THE ASSOCIATION
OF ENGINEERING
& SHIPBUILDING
DRAUGHTSMEN

AESD

ANALYSIS OF BRACED FRAMES by the Method of Shear Coefficients

SESSION 1958-59

By F. H. ABRAHAMS
A.M.I.Struct.E., M.Inst.W.

PRICE 3/6

THE ASSOCIATION OF ENGINEERING
AND SHIPBUILDING DRAUGHTSMEN

ANALYSIS OF BRACED FRAMES BY THE METHOD OF SHEAR COEFFICIENTS

by

F. H. ABRAHAMS, A.M.I.Struct.E., M.Inst.W.

Published by The Association of Engineering and Shipbuilding Draughtsmen, Onslow Hall, Little Green, Richmond, Surrey

SESSION 1958-59 9/59

SYNOPSIS.

ANALYSIS OF BRACED FRAMES BY THE METHOD OF SHEAR COEFFICIENTS.

By F. H. Abrahams, A.M.I.Struct.E., M.Inst.W.

Chief Designer, Braithwaite & Co. Structural Ltd., West Bromwich.
Lecturer in Design of Structural Steelwork at Dudley and Staffordshire Technical College.

INTRODUCTION.

The purpose of this pamphlet is to put forward a procedure for finding the loads in the members of certain types of braced frames by a simple and straightforward method which eliminates the labour of drawing reciprocal diagrams and influence lines. The method has been elaborated as a result of many years' experience as designer and lecturer, principally in consequence of the need for speed in analysis, although basically it is not new having been in use in America to some extent for many years, while one or two recently published English text-books also make limited reference to it. Generally speaking, however, this method does not appear to have received the attention in this country that it deserves.

It should be understood that the procedure is neither a "designer's dodge" nor an approximation. On the contrary it is scientifically and demonstrably sound, yielding exact results as well as saving time. It is in fact based upon the more usual methods of computation and may be described as a method of writing down directly on a space frame the results obtained diagrammatically while avoiding the labour of drawing the diagrams.

This does not mean that it is no longer necessary to learn how to draw reciprocal diagrams and influence lines. It simply means that having acquired a thorough knowledge of the underlying principles behind these diagrams it is possible to tabulate the results without drawing the diagrams.

In the case of dead loads the method is based upon the shear force and reciprocal diagrams, and in the case of live loads upon the influence lines. All these diagrams possess certain geometric properties and it is a proper understanding of the geometry of these diagrams that makes possible an intelligent use of the method of shear coefficients.

It should be added that the method is useful chiefly for the analysis of girders with parallel chords, since the additional calculation involved for girders with sloping chords probably cancels out the time saved on the diagrams.

BASIS OF PROCEDURE FOR DEAD LOAD.

Two commonly used types of frame—a Warren Girder and a Pratt Truss—will be taken to explain in detail the relationship between shear coefficients and reciprocal diagrams.

Fig. 1 illustrates the basis of procedure for a Warren Girder loaded at the lower chord panel points, unit loads being used for simplicity. The shear force diagram has been drawn in full and the reciprocal diagram for half of the girder.

The first point to note is that the values *s-a*, *s-b* and *s-c* in the reciprocal diagram equal the shears in panels A, B and C respectively and there is thus a close relationship between the loads in all the members and the shears in the respective panels. It is this relationship in fact which forms the basis of procedure using shear coefficients.

Consider now the reciprocal diagram in detail, commencing at the left-hand end. The reaction is represented by force line *a-s* and the loads in members A-1 and S-1 by force lines *a*-1 and *s*-1 respectively. Forces in A-1 and S-1 are thus multiples of the reaction, that is of the shear in the end panel, and the shear value of $2\frac{1}{2}$ is accordingly placed on these two members in the space frame.

For member 1-2 the reciprocal diagram shows that the load has the same value (but not the same sign) as for S-1 and this member also has the same shear coefficient of $2\frac{1}{2}$.

For members 2-3 and 3-4 reference to the reciprocal diagram shows that their loads are multiples of the shear in panel B, and they therefore have a shear coefficient of $1\frac{1}{2}$.

Similarly loads in members 4-5 and 5-6 are multiples of the shear in panel C and these members thus have a shear coefficient of $\frac{1}{2}$.

The diagonals in each panel therefore have shear coefficients equal to the shear in the respective panels, these shears being the vertical components of the loads in the diagonals.

Turning now to the chords, top chord member S-2 is represented in the reciprocal diagram by force line s-2 and the load in this member will clearly be double that in member A-1 since the slope of diagonal 1-2 is the same as that for S-1. Member S-2 therefore has a sheer coefficient of $2\frac{1}{2}+2\frac{1}{2}=5$. In other words, in passing from A-1 across 1-2 to S-2 the coefficient for S-2 is the sum of those for A-1 and 1-2. Expressed in another way, the load in S-2 equals the sum of the horizontal components of the loads in S-1 and 1-2.

Taking lower chord member B-3 next, its load exceeds that in S-2 by the effect of the slope of diagonal 2-3, that is by the horizontal component of the load in 2-3. Since member 2-3 has a coefficient of $1\frac{1}{2}$ the coefficient for B-3 will be the sum of these for S-2 and 2-3, that is $5+1\frac{1}{2}=6\frac{1}{2}$. Alternatively the coefficient for B-3 may

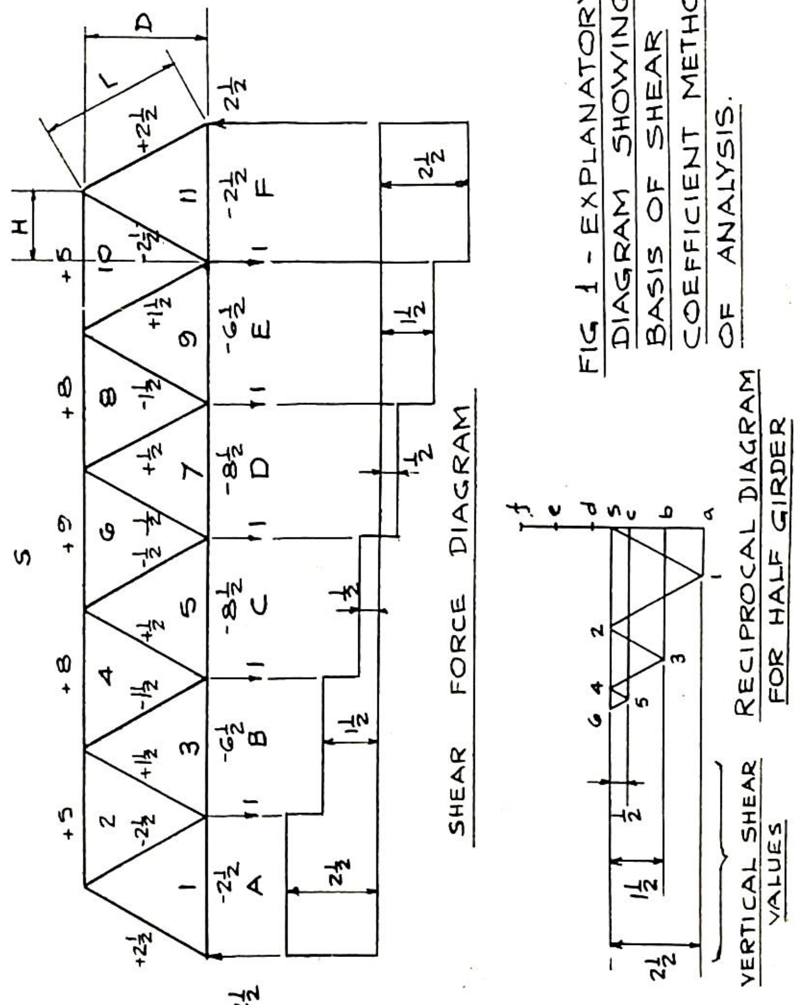


FIG 1 - EXPLANATORY DIAGRAM SHOWING BASIS OF SHEAR COEFFICIENT METHOD OF ANALYSIS.

be said to be the sum of these for A-1, 1-2 and 2-3, that is $2\frac{1}{2} + 2\frac{1}{2} + 1\frac{1}{2} = 6\frac{1}{2}$.

Similarly the load in S-4 exceeds that in B-3 by the horizontal component of diagonal 3-4, and its coefficient will be the sum of these for B-3 and 3-4, that is $6\frac{1}{2} + 1\frac{1}{2} = 8$. Alternatively the coefficient for S-4 will be the sum of those for S-2, 2-3 and 3-4, that is $5 + 1\frac{1}{2} + 1\frac{1}{2} = 8$.

In the same way the coefficient for C-5 will be the sum of those for S-4 and 4-5, that is $8 + \frac{1}{2} = 8\frac{1}{2}$, or alternatively the sum of those for B-3, 3-4 and 4-5, that is $6\frac{1}{2} + 1\frac{1}{2} + \frac{1}{2} = 8\frac{1}{2}$.

Finally the coefficient for S-6 will be the sum of those for C-5 and 5-6, that is $8\frac{1}{2} + \frac{1}{2} = 9$, or alternatively the sum of those for S-4, 4-5 and 5-6, that is $8 + \frac{1}{2} + \frac{1}{2} = 9$.

Coefficients for all members have now been found and the following general rules may be stated :—

1. Coefficients for the web members in each panel are the vertical shears in the panel.
2. Members connecting to the reaction have the same coefficient as the reaction, that is, the shear in the end panel.
3. Commencing at the reaction and proceeding towards the centre, panel by panel, the coefficients for the chord members add up across the diagonals.

For the sake of clarity the foregoing explanation commenced at the reactions, but in practice it is easier and more convenient to commence at the centre, as follows :—

Write down first the coefficients for the web members only, that is, 4-5 and 5-6 $= \frac{1}{2}$, 2-3 and 3-4 $= 1\frac{1}{2}$, and S-1 and 1-2 $= 2\frac{1}{2}$.

Starting now at the reaction and proceeding towards the centre, write down the coefficients for the chord members, that is, A-1 $= 2\frac{1}{2}$ (as for S-1), S-2 $= 2\frac{1}{2} + 2\frac{1}{2} = 5$, B-3 $= 5 + 1\frac{1}{2} = 6\frac{1}{2}$, S-4 $= 6\frac{1}{2} + 1\frac{1}{2} = 8$, C-5 $= 8 + \frac{1}{2} = 8\frac{1}{2}$, and S-6 $= 8\frac{1}{2} + \frac{1}{2} = 9$.

Signs attached to the coefficients indicate the nature of the loads, minus indicating tension and plus compression. Usually the nature of the loads may be seen by inspection, since the upper chord will be in compression and the lower chord will be in tension. If any doubt exists regarding the diagonals, a simple rule is that diagonals sloping downwards towards the centre of gravity of the loads, that is, the mid-point of the lower chord, are in tension and those sloping in the opposite direction are in compression. Thus members 1-2, 3-4 and 5-6 are in tension, and members S-1, 2-3 and 4-5 are in compression. This point may be checked by noting that the loads travel to the reaction by way of the web members. Thus the centre load hangs on diagonals 5-6 and 6-7, which share the load equally and must be in tension. Since these are in tension members 4-5 and 7-8 must be in compression for equilibrium. In the same way members 3-4, 8-9, 1-2 and 10-11 must be in tension, and members 2-3, 9-10, S-1 and S-11 must be in compression.

It can now be noted, also as a principle of general application, that if a section is cut through the frame at any position the algebraic summation of the shear coefficients for the chords and diagonals (ignoring the verticals) cut by the section line will always equal zero. Thus for a section cut through members S-2, 2-3 and B-3 the summation of coefficients is $+5+1\frac{1}{2}-6\frac{1}{2} = 0$.

It must be cleary understood that the shear coefficients placed against the members are not the loads in the members but are simply summations of vertical shears for unit panel point loads from which the actual loads can be calculated.

To find the loads consider first member A-1. Referring to triangle s-a-1 in the reciprocal diagram

$$\text{Load in A-1} = s\text{-}a \times \frac{a\text{-}1}{s\text{-}a}$$

But ratio $\frac{a\text{-}1}{s\text{-}a}$ in the reciprocal diagram = ratio $\frac{\mathrm{H}}{\mathrm{D}}$ in the space diagram.

$$\text{Hence load in A-1} = s\text{-}a \times \frac{\mathrm{H}}{\mathrm{D}} = 2\tfrac{1}{2} \times \frac{\mathrm{H}}{\mathrm{D}}.$$

The figure $2\frac{1}{2}$ is the shear coefficient for member A-1, H is the horizontal projection of diagonal S-1, and D is the vertical projection of the diagonal and also the depth of the girder.

If $\frac{\mathrm{H}}{\mathrm{D}}$ is called the Length Coefficient, then for any panel point load P (all panel loads being equal)

Load in A-1 = P × Shear Coefficient × Length Coefficient.

Consider now diagonal S-1. Referring again to triangle *s-a-*1 in the reciprocal diagram

$$\text{Load in S-1} = s\text{-}a \times \frac{s\text{-}1}{s\text{-}a} = s\text{-}a \times \frac{\mathrm{L}}{\mathrm{D}} = 2\tfrac{1}{2} \times \frac{\mathrm{L}}{\mathrm{D}}$$

L/D is the Length Coefficient for the diagonals, and hence for any panel point load P (all loads being equal)

Load in S-1 = P × Shear Coefficient × Length Coefficient.

For all chord members the length coefficient remains constant and equals $\frac{\mathrm{H}}{\mathrm{D}}$. Similarly for all diagonals the length coefficient equals $\frac{\mathrm{L}}{\mathrm{D}}$.

Thus it will be clear that once the shear coefficients have been written down on the space frame and the length coefficients calculated, the loads in the members can be tabulated at once providing the panel point loads are constant. Special cases with unequal panel point loads and with loads at alternate panel points only will be dealt with later.

Consider next the Pratt Truss shown in Fig. 2. Procedure generally will be similar to that for the warren girder, varied only by the shape of the frame which now includes verticals.

The vertical suspender 1-2 will have a coefficient of unity, since its sole function from the point of view of analysis is to transfer the panel point load to the upper chord where actually it acts, thus leaving members A-1 and B-2 a single continuous member.

Coefficients may be checked in detail against the shear force diagram and the reciprocal diagram in a similar manner to the Warren girder, and it should now be possible to write down the coefficients on the space frame.

Commencing at the centre of the girder, coefficients for web members corresponding to the shears in the respective panels are first inserted. The centre vertical is unloaded since load D-E passes directly into the two centre diagonals which will thus have a coefficient of $-\frac{1}{2}$, these bars being in tension. This half-ton load will pass into vertical 5-6 which will be in compression to equilibrate the vertical component of diagonal 6-7, and will thus have a coefficient of $+\frac{1}{2}$.

This load, joined by load C-D, will pass through diagonal 4-5 and vertical 3-4, giving them coefficients of $-1\frac{1}{2}$ and $+1\frac{1}{2}$ respectively. With load B-C added, diagonal 2-3 will have a coefficient of $-2\frac{1}{2}$, and with load A-B brought in the end post will have a coefficient of $+3\frac{1}{2}$ since this member must be in compression to equilibrate the load from suspender 1-2 and the vertical component of the load in diagonal 2-3. All these web coefficients, it will be seen, correspond to the shears in the respective panels.

Lower chord member A-1 and B-2 will have a coefficient of $-3\frac{1}{2}$ to equilibrate the horizontal component of the load in end post S-1, and following the same procedure as for the Warren girder and adding up across the diagonals, upper chord member S-3 will have a coefficient of $+6$, S-5 of $+7\frac{1}{2}$, and S-7 of $+8$. Similarly lower chord member C-4 will have a coefficient of -6, and D-6 of $-7\frac{1}{2}$.

It should be noted that the verticals have no effect in increasing the loads in the chords and this point will be made clear by inspection of the reciprocal diagram which explains that increments in the chord loads equal the horizontal components of the loads in the diagonals.

Reference to the reciprocal diagram and the space frame will show that the length coefficients follow the same general principle

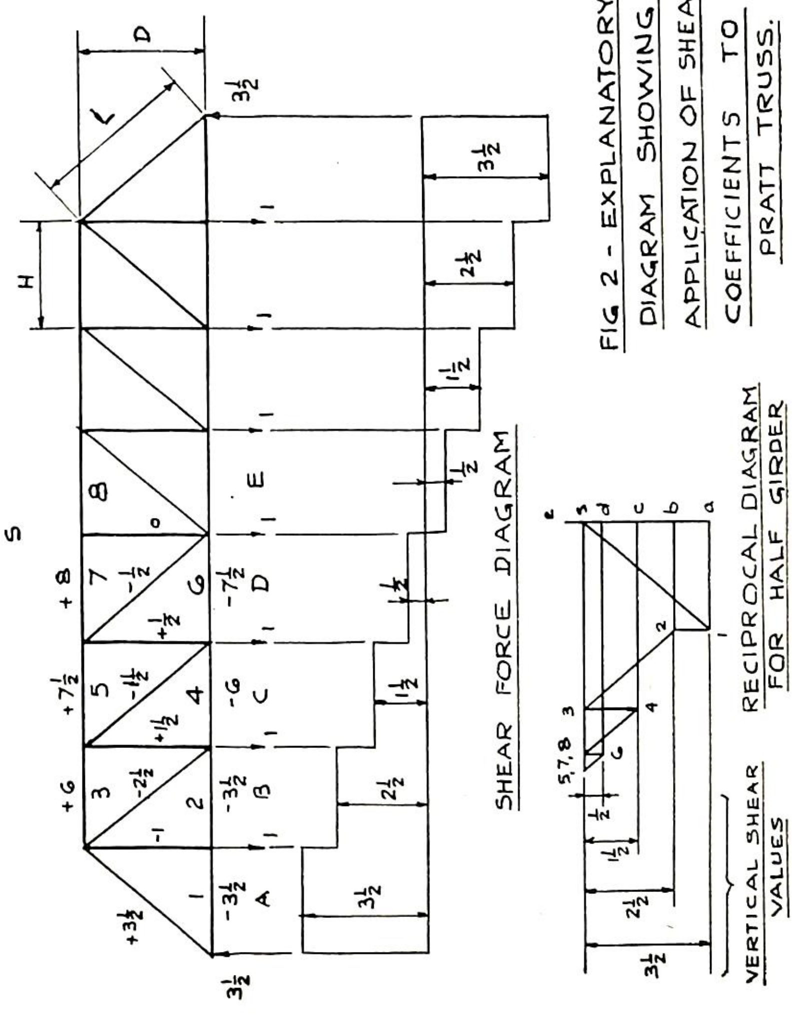


FIG 2 - EXPLANATORY DIAGRAM SHOWING APPLICATION OF SHEAR COEFFICIENTS TO PRATT TRUSS.

as for the Warren girder. The length coefficient for all the chord members will be the horizontal component of the diagonal slope divided by the vertical component, the latter being the depth of the girder. Similarly that for the diagonals will be the length of the diagonal divided by the depth of the girder, while that for the verticals will be the vertical length divided by the depth of the girder, which is unity.

From this it follows that loads in members for any panel point load P (all loads being equal) will be as follows :—

$$\text{Load in verticals} = \text{P} \times \text{shear coefficient.}$$

$$\text{Load in diagonals} = \text{P} \times \text{shear coefficient} \times \frac{\text{L}}{\text{D}}.$$

$$\text{Load in chords} = \text{P} \times \text{shear coefficient} \times \frac{\text{H}}{\text{D}}.$$

It should now be possible to take a few typical examples of commonly used frames to show how the method is applied in practice and how the loads in the members are tabulated.

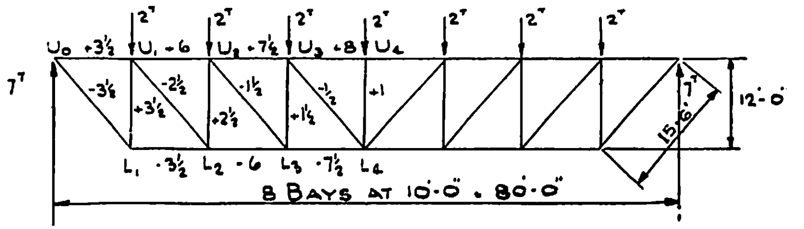


Fig. 3—Coefficients for Pratt truss, deck type, D = 12′ 0″, H = 10′ 0″, L = 15·6′, H/D = ·83, L/D = 1·3.

Fig. 3 shows a Pratt truss of Deck Type with a load of 2 tons at each upper chord panel point. Commencing at mid-span the shear coefficients based on unit loads are written down for the web members, and then working back from the reaction towards the centre the coefficients for the chord members are added on the principle of adding up across the diagonals. The coefficient for the centre vertical will be $+1$, since this member is a strut transferring the central load to the lower chord, where it divides, half going left and half right, giving a coefficient of $-\frac{1}{2}$ in the centre diagonals.

$$\text{Length coefficient for chord members} = \frac{10}{12} = 0{\cdot}83.$$

Length coefficient for diagonals $= \frac{15 \cdot 6}{12} = 1 \cdot 3.$

Length coefficient for verticals $=$ unity.

Loads in members may now be tabulated.

Vertical	U4-L4	=	$+(2\times 1)$	=	$+2$ tons.
	U3-L3	=	$+(2\times 1\frac{1}{2})$	=	$+3$ tons.
	U2-L2	=	$+(2\times 2\frac{1}{2})$	=	$+5$ tons.
	U1-L1	=	$+(2\times 3\frac{1}{2})$	=	$+7$ tons.
Diagonal	U3-L4	=	$-(2\times \frac{1}{2}\times 1\cdot 3)$	=	$-1\cdot 3$ tons.
	U2-L3	=	$-(2\times 1\frac{1}{2}\times 1\cdot 3)$	=	$-3\cdot 9$ tons.
	U1-L2	=	$-(2\times 2\frac{1}{2}\times 1\cdot 3)$	=	$-6\cdot 5$ tons.
	U0-L1	=	$-(2\times 3\frac{1}{2}\times 1\cdot 3)$	=	$-9\cdot 1$ tons.
Upper Chord	U0-U1	=	$+(2\times 3\frac{1}{2}\times 0\cdot 83)$	=	$+5\cdot 81$ tons.
	U1-U2	=	$+(2\times 6\times 0\cdot 83)$	=	$+9\cdot 96$ tons.
	U2-U3	=	$+(2\times 7\frac{1}{2}\times 0\cdot 83)$	=	$+12\cdot 45$ tons.
	U3-U4	=	$+(2\times 8\times 0\cdot 83)$	=	$+13\cdot 28$ tons.
Lower Chord	L1-L2	=	$-(2\times 3\frac{1}{2}\times 0\cdot 83)$	=	$-5\cdot 81$ tons.
	L2-L3	=	$-(2\times 6\times 0\cdot 83)$	=	$-9\cdot 96$ tons.
	L3-L4	=	$-(2\times 7\frac{1}{2}\times 0\cdot 83)$	=	$-12\cdot 45$ tons.

The simplest method of writing down these loads, using a slide rule, is to combine the panel point load and the length coefficient into a combined coefficient and then to multiply by the shear coefficient. With this procedure it is not necessary to write out a table of loads. The shear coefficients can be written on one half of the space frame, and the loads calculated by slide rule on the other half of the frame, the whole operation being carried through in a few minutes.

Fig. 4 shows a Pratt truss loaded on both upper and lower chords, a condition commonly found in bridge design. The procedure is generally as before but it is desirable, to avoid too much complication, to take the upper and lower chord load systems separately in the manner shown. Alternatively the coefficients for the two systems can be written on the same diagram using different coloured pencils.

The centre diagonals will be unloaded for both loading systems since there is no shear in this panel. Vertical U1-L1 will be unloaded for upper chord loads, these loads passing to the reaction directly through the end post. Vertical U3-L3 will be unloaded for lower chord loads, which pass to the reaction through web members U2-L3, etc.

Taking the upper chord loads first, the coefficients for the web members correspond to the shears due to unit loads and verticals U3-L3 and U2-L2 together with end post U1-L0 must be in compression, and hence diagonals U2-L3 and U1-L2 (sloping towards

the centre) will be in tension. The chord coefficient will follow the same procedure as before.

For the lower chord loads, diagonals U2-L3 and U1-L2 must be in tension since the loads hang on these members, so that vertical U2-L2 and end post U1-L0 will be in compression. The chord coefficients will again follow standard procedure.

Comparing the two sets of coefficients it will be seen that those for the verticals differ for the two load systems, but for all other members the coefficients are the same for both systems.

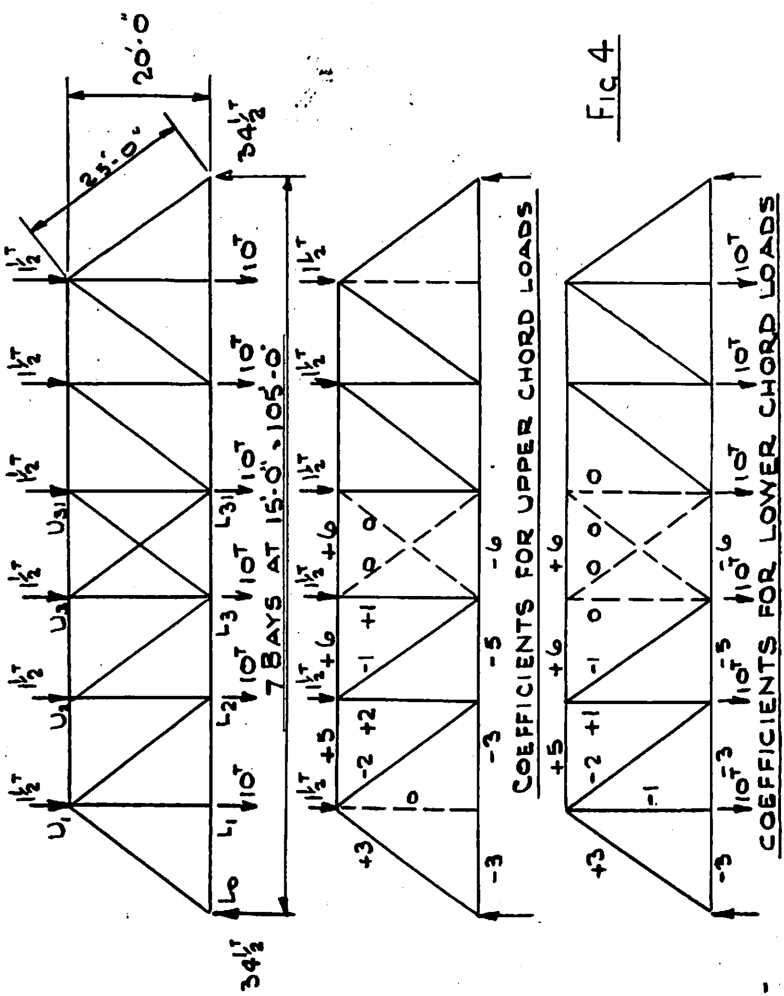


FIG 4

Length coefficient for chord members $= \dfrac{15}{20} = 0{\cdot}75.$

Length coefficient for diagonals $= \dfrac{25}{20} = 1{\cdot}25.$

Length coefficient for verticals = unity.

Load in members due to upper chord loading system.

Vertical	U3-L3	=	$+(1\frac{1}{2}\times 1)$	=	$+1{\cdot}5$ tons.
	U2-L2	=	$+(1\frac{1}{2}\times 2)$	=	$+3$ tons.
	U1-L1				Nil.
Diagonal	U2-L3	=	$-(1\frac{1}{2}\times 1\times 1{\cdot}25)$	=	$-1{\cdot}88$ tons.
	U1-L2	=	$-(1\frac{1}{2}\times 2\times 1{\cdot}25)$	=	$-3{\cdot}75$ tons.
Centre Diagonals					Nil.
End Post	U1-L0	=	$+(1\frac{1}{2}\times 3\times 1{\cdot}25)$	=	$+5{\cdot}63$ tons.
Upper Chord	U1-U2	=	$+(1\frac{1}{2}\times 5\times 0{\cdot}75)$	=	$+5{\cdot}63$ tons.
	U2-U3	=	$+(1\frac{1}{2}\times 6\times 0{\cdot}75)$	=	$+6{\cdot}75$ tons.
	U3-$U3_1$	=	$+(1\frac{1}{2}\times 6\times 0{\cdot}75)$	=	$+6{\cdot}75$ tons.
Lower Chord	L0-L2	=	$-(1\frac{1}{2}\times 3\times 0{\cdot}75)$	=	$-3{\cdot}38$ tons.
	L2-L3	=	$-(1\frac{1}{2}\times 5\times 0{\cdot}75)$	=	$-5{\cdot}63$ tons.
	L3-$L3_1$	=	$-(1\frac{1}{2}\times 6\times 0{\cdot}75)$	=	$-6{\cdot}75$ tons.

Loads in members due to lower chord loading system.

Vertical	U3-L3	=			Nil.
	U2-L2	=	$+(10\times 1)$	=	$+10$ tons.
	U1-L1	=	$-(10\times 1)$	=	-10 tons.
Diagonal	U2-L3	=	$-(10\times 1\times 1{\cdot}25)$	=	$-12{\cdot}5$ tons.
	U1-L2	=	$-(10\times 2\times 1{\cdot}25)$	=	-25 tons.
Centre Diagonals		=			Nil.
End Post	U1-L0	=	$+(10\times 3\times 1{\cdot}25)$	=	$+37{\cdot}5$ tons.
Upper Chord	U1-U2	=	$+(10\times 5\times 0{\cdot}75)$	=	$+37{\cdot}5$ tons.
	U2-U3	=	$+(10\times 6\times 0{\cdot}75)$	=	$+45$ tons.
	U3-$U3_1$	=	$+(10\times 6\times 0{\cdot}75)$	=	$+45$ tons.
Lower Chord	L0-L2	=	$-(10\times 3\times 0{\cdot}75)$	=	$-22{\cdot}5$ tons.
	L2-L3	=	$-(10\times 5\times 0{\cdot}75)$	=	$-37{\cdot}5$ tons.
	L3-$L3_1$	=	$-(10\times 6\times 0{\cdot}75)$	=	-45 tons.

The total load in each member will, of course, be the sum of the loads due to the two systems.

Having written down separately the loads due to the two systems it will be clear that these can be combined with a single calculation. For example :—

Vertical U2-L2 $= +(1\frac{1}{2}\times 2) + (10\times 1) = +13$ tons.

Upper Chord U1-U2 $= +(11\frac{1}{2}\times 5\times 0{\cdot}75) = +43{\cdot}13$ tons.

Fig. 5 shows a Pratt truss with sub-divided bays such as is commonly used for longer spans, cross girders being located both at the main verticals and at the intermediate verticals, the latter serving solely as hangers to transfer the load to centre intersection points C1, C3, C5 and C7. Compared with previous examples these additional loads are the only complication and an understanding of the manner in which they are transferred to the reactions will resolve this difficulty.

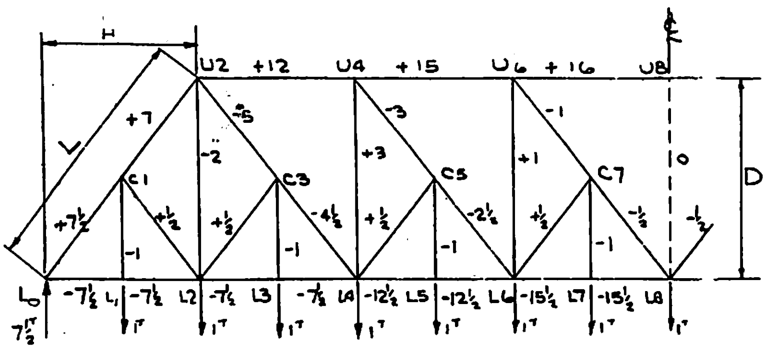


Fig. 5—Coefficients for Pratt truss with sub-divided bays.

The load at C7 is shared equally between C7-U6 and C7-L6, the half in C7-U6 passing to the reaction via U6-L6, L6-U4, U4-L4, L4-U2 and U2-L0, while the half in C7-L6 passes to the reaction via L6-U4, U4-L4, L4-U2, and U2-L0, that is to say the sub-vertical loads pass into the main web members.

Similarly the load at C5 is shared equally between C5-U4 and C5-L4, the half in C5-U4 passing to the reaction via U4-L4, L4-U2, and U2-L0, while the half in C5-L4 passes to the reaction via L4-U2 and U2-L0.

In the same way the load at C3 is shared equally by C3-U2 and C3-L2, the half in C3-U2 passing down U2-L0 to the reaction, while the half in C3-L2 passes to the reaction via L2-U2 and U2-L0.

The load at C1 is shared equally between C1-L0 and C1-L2, the half in C1-L2 passing to the reaction via L2-U2 and U2-L0.

It therefore follows that all sub-verticals will be in tension with a coefficient of -1, while all sub-diagonals will be in compression with a coefficient of $+\frac{1}{2}$.

The combined coefficients for the main web members can best be written down commencing at the centre. L8-C7 will have a coefficient of $-\frac{1}{2}$ as in previous examples, this being the shear in this panel, and this will be increased at C7 by half the sub-vertical

load to give a coefficient of -1 in C7-U6. The load from C7-U6 will pass into U6-L6, which will thus have a coefficient of $+1$, this member being in compression to equilibrate the vertical component of C7-U6.

At L6 the load from U6-L6 will be joined by the half load from C7-L6 and by the load hanging at L6 to give a coefficient of $-2\frac{1}{2}$ in L6-C5 to correspond to the shear in this panel. At C5 the load from L6-C5 is increased by half the sub-vertical load to give a coefficient of -3 in C5-U4, this load in turn passing into U4-L4 which will thus have a coefficient of $+3$.

At L4 the load from U4-L4 will be joined by the half load from C5-L4 and by the load hanging at L4 to give a coefficient of $-4\frac{1}{2}$ in L4-C3, which again equals the shear in this panel. At C3 the load from L4-C3 is increased by half the sub-vertical load to give a coefficient of -5 in C3-U2.

Vertical U2-L2 supports the load hanging at L2 and also receives the half loads from C3-L2 and C1-L2, thus having a total coefficient of -2.

End post U2-C1 equilibrates the loads in U2-L2 and C3-U2, giving a coefficient of $+7$ for this member, and this is increased at C1 by the half load from the sub-vertical to give a coefficient of $+7\frac{1}{2}$ for C1-L0, which equals the shear in the end panel.

Once the above general principle is understood the coefficients for the web members will be found to follow a fixed principle. Coefficients for all sub-verticals will be -1, those for all sub-diagonals $+\frac{1}{2}$, and that for vertical U2-L2 will be -2. Coefficients for the lower portions of main diagonals U6-L8, U4-L6 and U2-L4 equal the shear in the respective panels, and these are increased by $\frac{1}{2}$ for the upper portions of the diagonals. Coefficients for verticals U6-L6 and U4-L4 are the same as for the upper portion of the adjacent diagonal. The coefficient for the upper portion of the end post will always equilibrate U2-L2 and U2-C3, while that for the lower portion will equal the shear in the end panel.

Coefficients for the upper chord members add up across the upper half of the diagonals, following the procedure described previously. Hence U2-U4 will have a coefficient of $+12$ to equilibrate U2-C1 and U2-C3, U4-U6 will have a coefficient of $+15$ to equilibrate U2-U4 and U4-C5, and U6-U8 will have a coefficient of $+16$ to equilibrate U4-U6 and U6-C7.

For the lower chord, L0-L2 will have a coefficient of $-7\frac{1}{2}$ corresponding to the lower portion of the end post, and L2-L4 will also have a coefficient of $-7\frac{1}{2}$ since the horizontal components of C1-L2 and C3-L2 are equal and in opposition to each other. L4-L6 will have a coefficient of $-12\frac{1}{2}$ to equilibrate L2-L4, C3-L4 and C5-L4, and L6-L8 will have a coefficient of $-15\frac{1}{2}$ to equilibrate L4-L6, C5-L6 and C7-L6.

The coefficients may be checked by the method of sections as previously explained. If a section is cut through the frame at any point the algebraic sum of the coefficients for the chords and the diagonals cut through (ignoring the verticals) must equal zero. For example a section slightly to the right of L0-U2 gives a summation of $-7\frac{1}{2}+\frac{1}{2}-5+12 = 0$. A section to the right of U4-L4 gives a summation of $-12\frac{1}{2}+\frac{1}{2}-3+15 = 0$. A section drawn through the frame to the right of points L4 and U6 gives a summation of $-12\frac{1}{2}-2\frac{1}{2}-1+16 = 0$, and so on.

Having found the shear coefficients the loads in the members due to any given panel point load may be calculated by the method previously described, the length coefficient for the sub-diagonals being the same as for the main diagonals and those for the sub-verticals being unity.

A truss of this size will also have dead loads at the upper chord panel points. These should be dealt with by the method given for Fig. 4, ignoring the sub-verticals and sub-diagonals completely since these members will not be effected by loads at the upper chord panel points (see the next example for the method of dealing with such loads).

Fig. 6 gives the procedure for a Warren girder with sub-divided bays, which varies from the preceding example only by virtue of the shape of the truss. Sub-verticals will have a coefficient of -1 and sub-diagonals of $+\frac{1}{2}$. Verticals U6-L6 and U2-L2 will each have a coefficient of -2 as in the case of U2-L2 in the previous example and for the same reason. Verticals U8-L8 and U4-L4 are unloaded from lower chord loads. Diagonal C7-L8 will have a coefficient of $-\frac{1}{2}$ as in the previous example and with sub-vertical

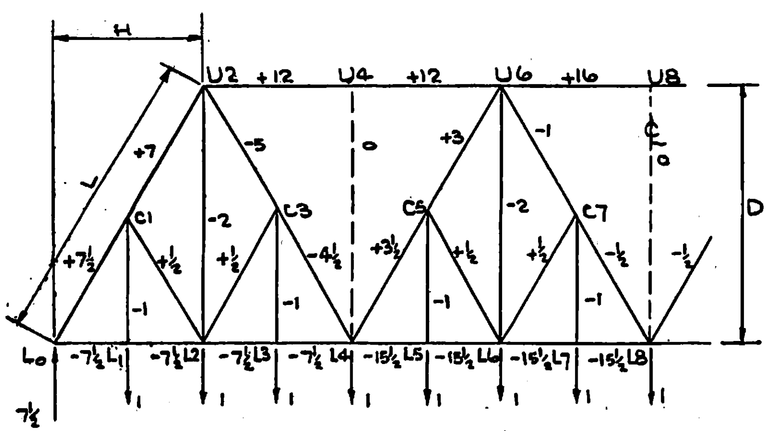


Fig. 6—Coefficients for Warren girder with sub-divided bays.

load added at C7 diagonal C7-U6 will have a coefficient of -1, also as before, Diagonal U6-C5 must have a coefficient of $+3$ to equilibrate C7-U6 and U6-L6 and with the sub-vertical load added at C5 diagonal C5-L4 will have a coefficient of $+3\frac{1}{2}$, which is the shear in the panel. Diagonal C3-L4 must have a coefficient of $-4\frac{1}{2}$ (the shear in the panel) to equilibrate C5-L4 and the load hanging at L4, and with half the sub-vertical load entering the system at C3, diagonal C3-L2 will have a coefficient of -5. U2-C1 must have a coefficient of $+7$ to equilibrate C3-U2 and U2-L2, and with half the sub-vertical load added at C1, member C1-L0 has a coefficient of $+7\frac{1}{2}$, the shear in the end panel.

Chord coefficients again add up across the diagonals, so that member U2-U6 has a coefficient of $+12$ corresponding to the horizontal components of U2-C1 and U2-C3, and similarly member U6-U8 has a coefficient of $12+3+1 = 16$. For the lower chord, member L0-L4 has a coefficient of $-7\frac{1}{2}$ as in Fig. 5, and member L4-L8 a coefficient of $-15\frac{1}{2}$ to equilibrate L0-L4, C3-L4 and L4-C5.

If a section is drawn anywhere across the frame it will be found that the summation of the coefficient in the chords and diagonals equals zero as in previous cases.

Trusses of this type will also be loaded on the upper chord, and for unit loads at U2, U4, U6 and U8, the coefficients will be as follows :—

Verticals	U8-L8 and U4-L4		$+1$
	U6-L6 and U2-L2		Nil.
Diagonal	L8-U6		$-\frac{1}{2}$
	U6-L4		$+1\frac{1}{2}$
	L4-U2		$-2\frac{1}{2}$
End post	U2-L0		$+3\frac{1}{2}$
Lower Chord	L0-L4		$-3\frac{1}{2}$
	L4-L8	$-(3\frac{1}{2}+2\frac{1}{2}+1\frac{1}{2}) =$	$-7\frac{1}{2}$
Upper Chord	U2-U6	$+(3\frac{1}{2}+2\frac{1}{2}) =$	$+6$
	U6-U8	$+(6+1\frac{1}{2}+\frac{1}{2}) =$	$+8$
Sub-verticals and sub-diagonals			Nil.

In certain cases loads occur only at alternate panel points as in the case of the lattice roof girder shown in Fig. 7. Procedure is generally as before with web coefficients corresponding to the shears and chord coefficients adding up across the diagonals. The procedure should be self-explanatory and the load in each member will be P × shear coefficient × length coefficient.

Occasionally it happens that one of the loads supported by a frame is larger than all the others, or a frame may be required to support a runway load in addition to the normal loading. In the first case the general principle is to find first the loads in the mem-

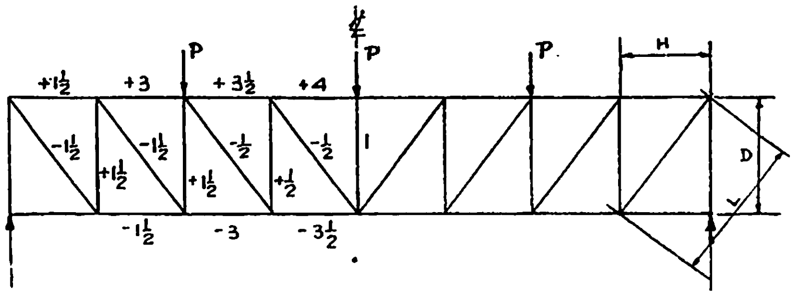


Fig. 7—Coefficients for roof girder with trusses at alternate panel points.

bers due to equal panel loads, and then the additional loads due to the excess load at one panel point. In the second case the normal loading is taken first and the runway load separately.

Fig. 8 is a typical example of a roof girder supporting a runway. Coefficients for the roof loads follow normal procedure, but those for the runway load on the lower chord require some explanation.

The shear in the left-hand panel (for unit load) is $\frac{5}{8}$ and in the right hand panel $\frac{3}{8}$. These shears give the coefficients for all web members and since the load hangs at L3 members U2-L3 and U3-L3 must both be in tension and have coefficients of $-\frac{5}{8}$ and $-\frac{3}{8}$ respectively. This will automatically fix the signs for all web coefficients, since in each case the vertical must equilibrate the vertical component of the diagonal. Chord coefficients will again add up across the diagonals and as the loading is unbalanced it is desirable to add up from both ends. As a check on these figures a section may be cut through the frame to the right of U3-L3 giving a coefficient summation of $-1\frac{7}{8}+\frac{3}{8}+1\frac{1}{2} = 0$, which is correct.

With an unbalanced load on the frame the total load in corresponding members on each side of the centre line will not be the same, and since only the maximum design load is required the worst case only will be taken. Since this is a special case the design loads are tabulated in detail below.

Length of diagonals $= \sqrt{10^2+8^2} = 12{\cdot}8$ feet.

Diagonal length coefficient $= 12{\cdot}8/8 = 1{\cdot}6$.

Chord length coefficient $= 10/8 = 1{\cdot}25$.

Vertical length coefficient $=$ unity.

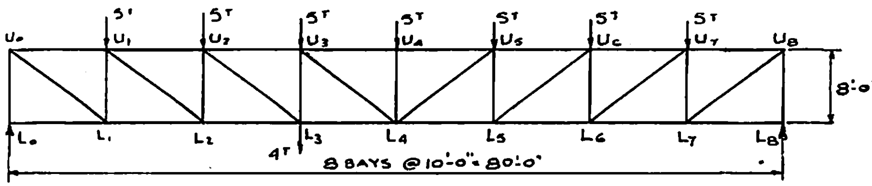


FIG 8 ROOF GIRDER WITH RUNWAY ON LOWER CHORD

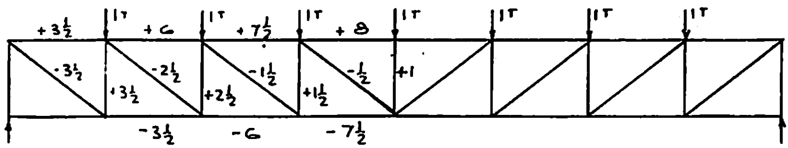


FIG 8(a) - COEFFICIENTS FOR UNIT LOADS ON UPPER CHORD

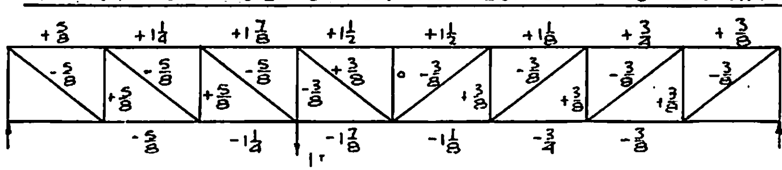


FIG 8(b) - COEFFICIENTS FOR UNIT LOAD AT JOINT L3.

Maximum Loads.

Verticals	U1-L1, U7-L7	=	$+(3\frac{1}{2}\times 5)+(\frac{5}{8}\times 4)$	$=$ +20 tons.
	U2-L2, U6-L6	=	$+(2\frac{1}{2}\times 5)+(\frac{5}{8}\times 4)$	$=$ +15 tons.
	U3-L3, U5-L5	=	$+(1\frac{1}{2}\times 5)+(\frac{3}{8}\times 4)$	$=$ + 9 tons.
	U4-L4	=	$+(1\times 5)$	$=$ + 5 tons.
Diagonals	U0-L1, U8-L7	=	$-1{\cdot}6\,\{(3\frac{1}{2}\times 5)+(\frac{5}{8}\times 4)\}$	$=$ −32 tons.
	U1-L2, U7-L6	=	$-1{\cdot}6\,\{(2\frac{1}{2}\times 5)+(\frac{5}{8}\times 4)\}$	$=$ −24 tons.
	U2-L3, U6-L5	=	$-1{\cdot}6\,\{(1\frac{1}{2}\times 5)+(\frac{5}{8}\times 4)\}$	$=$ −16 tons.
	U3-L4, U5-L4	=	$-1{\cdot}6\,\{(\frac{1}{2}\times 5)+(\frac{3}{8}\times 4)\}$	$=$ −6·4 tons.
Upper Chord	U0-U1, U8-U7	=	$+1{\cdot}25\,\{(3\frac{1}{2}\times 5)+(\frac{5}{8}\times 4)\}$	$=$ +25 tons.
	U1-U2, U7-U6	=	$+1{\cdot}25\,\{(6\times 5)+(1\frac{1}{4}\times 4)\}$	$=$ +43¾ tons.
	U2-U3, U6-U5	=	$+1{\cdot}25\,\{(7\frac{1}{2}\times 5)+(1\frac{7}{8}\times 4)\}$	$=$ +56¼ tons.
	U3-U4, U5-U4	=	$+1{\cdot}25\,\{(8\times 5)+(1\frac{1}{2}\times 4)\}$	$=$ +57½ tons.

Lower Chord L1-L2, L7-L6 $= -1{\cdot}25\{(3\frac{1}{2}\times 5)+(\frac{5}{8}\times 4)\} = -25$ tons.

L2-L3, L6-L5 $= -1{\cdot}25\{(6\times 5)+(1\frac{1}{4}\times 4)\} = -43\frac{3}{4}$ tons.

L3-L4, L5-L4 $= -1{\cdot}25\{(7\frac{1}{2}\times 5)+(1\frac{7}{8}\times 4)\} = -56\frac{1}{4}$ tons.

It will be seen that for corresponding members on each side of the centre line the lesser value of the coefficient due to runway load has been ignored, while for corresponding members which have coefficients of opposite signs—as for example U3-L3 and U5-L5—the worst design case has been taken, since this is the only condition that matters.

When an exceptional case occurs in which the runway load is very large compared with the roof loads, it may become necessary to tabulate the load in every member of the frame. For instance, with a runway load of 20 tons and roof loads remaining at 5 tons, the following conditions for corresponding members U3-L3 and U5-L5 must be taken into account.

U3-L3, with 20 ton load fully operative
load $= +(1\frac{1}{2}\times 5) - (\frac{3}{8}\times 20 = 0$
with 20 ton load inoperative
load $= +(1\frac{1}{2}\times 5) = +7\frac{1}{2}$ tons.

U5-L5, with 20 ton load fully operative
load $= +(1\frac{1}{2}\times 5) + (\frac{3}{8}\times 20) = +15$ tons.

Similar alternatives occur with diagonals U3-L4 and U5-L4.

U3-L4, with 20 ton load fully operative
load $= \{-(\frac{1}{2}\times 5) + (\frac{3}{8}\times 20)\}\times 1{\cdot}6 = +8$ tons
with 20 ton load inoperative
load $= -(\frac{1}{2}\times 5\times 1{\cdot}6) = -4$ tons.

U5-L4, with 20 ton load fully operative
load $= -1{\cdot}6\{(\frac{1}{2}\times 5) + (\frac{3}{8}\times 20)\} = -16$ tons.

It may happen occasionally that a special wide panel with unequal loading is required, and an understanding of the basic principles behind the method of shear coefficients will enable a quick and accurate analysis to be made without the necessity of drawing a reciprocal diagram. Fig. 9 shows a typical case and after previous explanations the analysis below should be clear without a detailed discussion.

Shears in panels.

End panel	10·8 tons.
Second panel	6·8 tons.
Third panel	2·4 tons.

Diagonal lengths.

U0-L1, U1-L2 $= 8\sqrt{2}$ feet.

U2-L3 $= \sqrt{10^2+8^2} = 12{\cdot}8$ feet.

Diagonal length coefficients.

U0-L1, U1-L2 $= \sqrt{2}$

U2-L3 $=$ 12·8/8 $= 1{\cdot}6$

Vertical lengths coefficients $=$ unity

Chord length coefficients.

All chords except U2-U3 $= 8/8 = 1$

U2-U3 $= 10/8 = 1{\cdot}25$

(*Note.*—Length coefficient for L2-L3 is the same as for U0-U1, U1-U2 and L1-L2, because the load in L2-L3 is based upon the slope of diagonal U1-L2 and not U2-L3).

Loads in verticals.

U1-L1 = shear in end panel $= +10{\cdot}8$ tons.

U2-L2 = shear in second panel $= +\ 6{\cdot}8$ tons.

U3-L3 $= +\ 4{\cdot}8$ tons.

Loads in diagonals.

U0-L1 $= -10{\cdot}8 \times \sqrt{2} = -15{\cdot}27$ tons.

U1-L2 $= -6{\cdot}8 \times \sqrt{2} = -\ 9{\cdot}61$ tons.

U2-L3 $= -2{\cdot}4 \times 1{\cdot}6 = -\ 3{\cdot}84$ tons.

Loads in chords.

U0-U1 $= +10{\cdot}8 \times 1 = +10{\cdot}8$ tons.

L1-L2 $= -10{\cdot}8 \times 1 = -10{\cdot}8$ tons.

U1-U2 $= +10{\cdot}8 +$ horizontal component of U1-L2

$= +10{\cdot}8 + 6{\cdot}8 = +17{\cdot}6$ tons.

L2-L3 $= -17{\cdot}6$ tons.

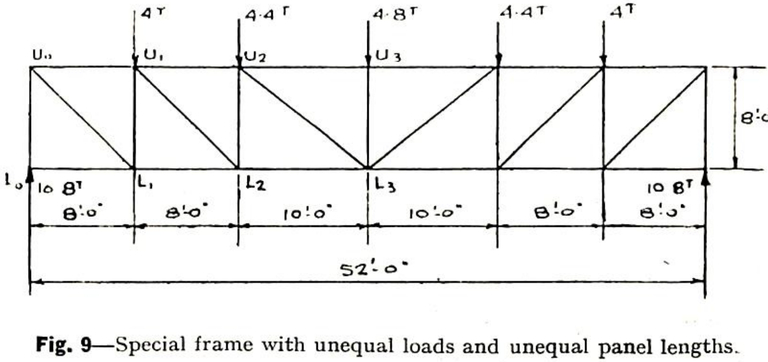


Fig. 9—Special frame with unequal loads and unequal panel lengths.

$$\text{U2-U3} = +17{\cdot}4 + \text{horizontal component of U2-L3}$$

$$= +17{\cdot}4 + \left(3{\cdot}84 \times \frac{10}{12{\cdot}8}\right) = +20{\cdot}4 \text{ tons.}$$

$$\text{Or} \quad = +17{\cdot}4 + \left(2{\cdot}4 \times \frac{10}{8}\right) = +20{\cdot}4 \text{ tons.}$$

In a special frame of this kind it may be found more convenient to find the loads in chord members by reference to the bending moment. Thus the load in U2-U3 equals the moment at L3 divided by the depth, and the load in U1-U2 and L2-L3 equals the moment at L2 divided by the depth.

BASIS OF PROCEDURE FOR LIVE LOADS.

Procedure generally is similar to that for dead load with the important difference that under live load all web members except those in the end panels will have a reversal of load, that is to say they will sometimes be in compression and sometimes in tension according to the positions of the load on the span. In the case of the chords, the upper chord will always be in compression and the lower chord always in tension for vertical live loads, and the maximum values of chord loads (in every panel) will occur with the span fully loaded. This condition is exactly the same as for dead load, and live loads coefficients for all chord members will be the same as for dead load, and can be copied from the dead load diagram. What is required therefore is a system for finding the maximum compression and maximum tension in each web member, and it will be found that the method of shear coefficients achieves this quite simply without the labour of drawing influence diagrams, and is in fact a method of writing down directly the results obtained from influence diagrams.

Consider the case of the Warren girder shown in Figs. 10 (*a*) - (*f*). A single load is shown moving across the girder from left to right, and the shear coefficients are given with the load at each panel point in turn following the procedure already explained for dead loads. With the moving load at L1 loads in all members correspond to the coefficients in Fig. 10 (*a*), with the load at L2 loads correspond to coefficients in Fig. 10 (*b*), and so on. With the load at L1 member U1-L1 has a negative coefficient of $\frac{1}{7}$, which is the highest negative coefficient this member can have. With the load at L2 member U2-L2 has a negative coefficient of $^2/_7$, and with a load at both L1 and L2 member U2-L2 has a total negative coefficient of $\frac{3}{7}$, which is the highest negative coefficient this member can have. Similarly with loads at L1, L2 and L3, member U3-L3

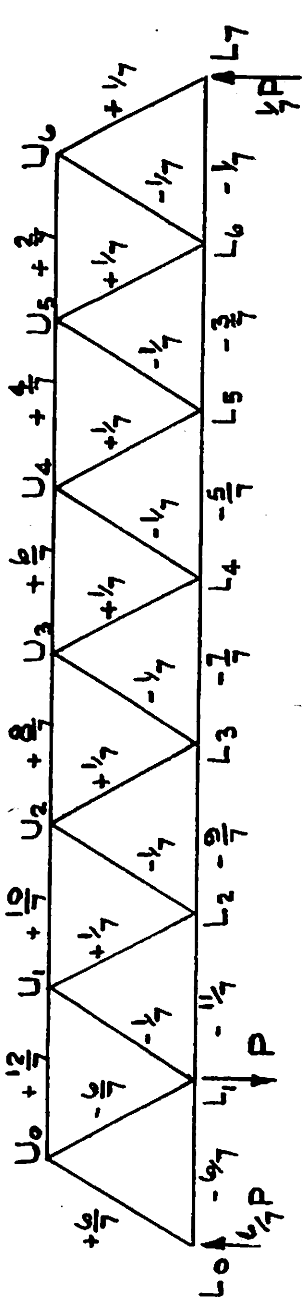


FIG-10(a) COEFFICIENTS FOR SINGLE ROLLING LOAD (PLACED AT L1)

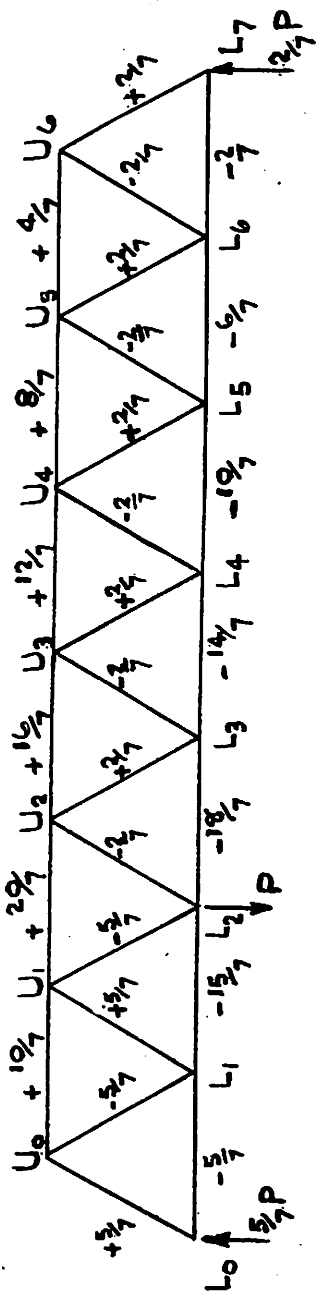


FIG-10(b) COEFFICIENTS FOR SINGLE ROLLING LOAD (PLACED AT L2)

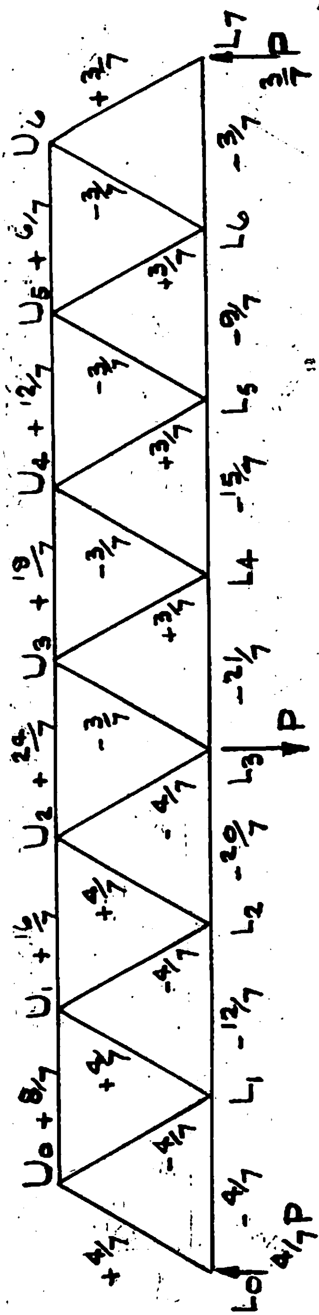


FIG 10(c) COEFFICIENTS FOR SINGLE ROLLING LOADS (PLACED AT L3)

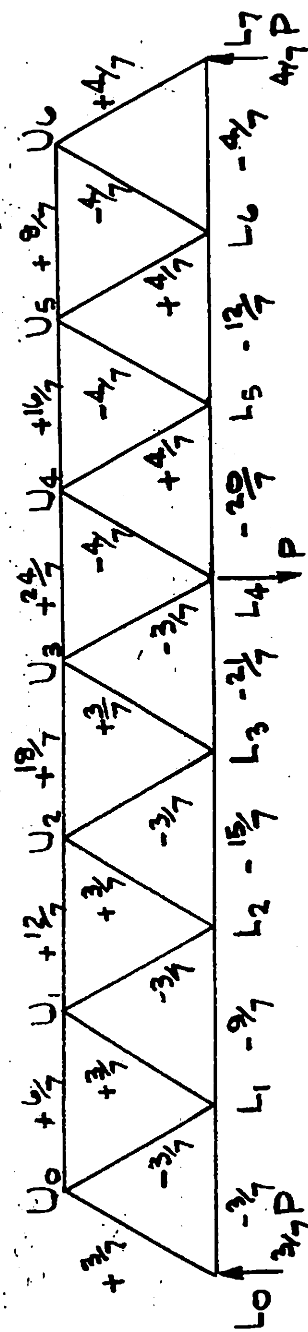


FIG 10(d) COEFFICIENTS FOR SINGLE ROLLING LOAD (PLACED AT L4)

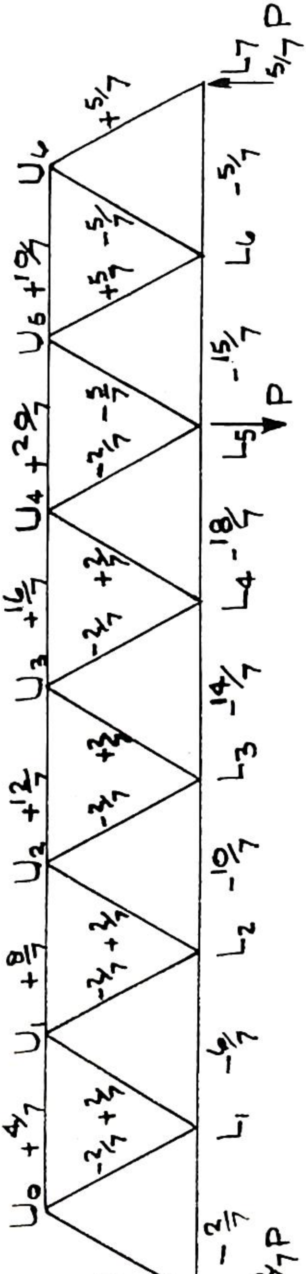


FIG-10(e) COEFFICIENTS FOR SINGLE ROLLING LOAD (PLACED AT L5)

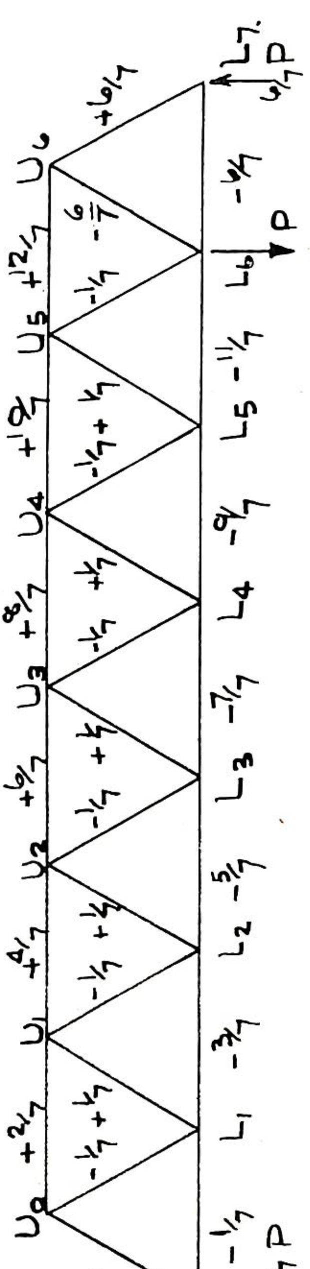
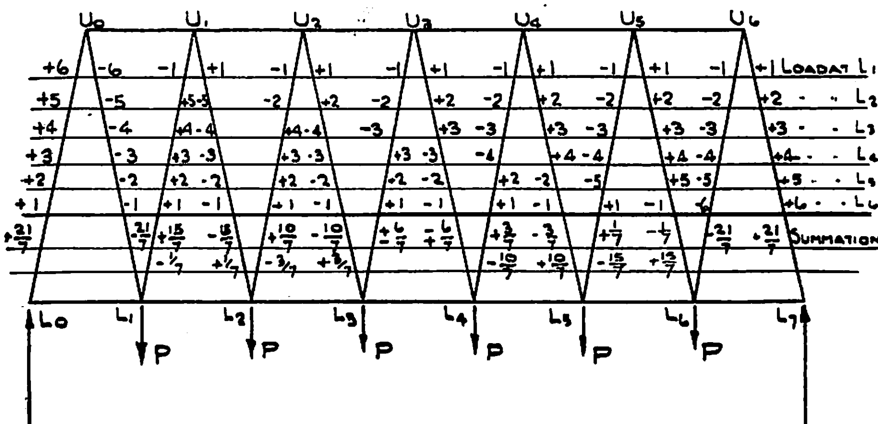


FIG-10(f) COEFFICIENTS FOR SINGLE ROLLING LOAD (PLACED AT L6)

+ 2	+ 4	+ 6	+ 8	+ 10	+ 12	For load at L6
+ 4	+ 8	+ 12	+ 16	+ 20	+ 10	L5
+ 6	+ 12	+ 18	+ 24	+ 16	+ 8	L4
+ 8	+ 16	+ 24	+ 18	+ 12	+ 6	L3
+ 10	+ 20	+ 16	+ 12	+ 8	+ 4	L2
+ 12	+ 10	+ 8	+ 6	+ 4	+ 2	L1
$+\frac{42}{7}$	$+\frac{70}{7}$	$+\frac{84}{7}$	$+\frac{84}{7}$	$+\frac{70}{7}$	$+\frac{42}{7}$	Summations



- 6	- 11	- 9	- 7	- 5	- 3	- 1	Load at L_1
- 5	- 15	- 18	- 14	- 10	- 6	- 2	L_2
- 4	- 12	- 20	- 21	- 15	- 9	- 3	L_3
- 3	- 9	- 15	- 21	- 20	- 12	- 4	L_4
- 2	- 6	- 10	- 14	- 18	- 15	- 5	L_5
- 1	- 3	- 5	- 7	- 9	- 11	- 6	L_6
$-\frac{21}{7}$	$-\frac{56}{7}$	$-\frac{77}{7}$	$-\frac{84}{7}$	$-\frac{77}{7}$	$-\frac{56}{7}$	$-\frac{21}{7}$	Summations

FIG-11 SUMMATIONS OF COEFFICIENTS FOR LOADING CONDITIONS IN FIGS-10(a) TO (f) SHEWING MAX & MIN COEFFICIENTS IN WEB MEMBERS

has its maximum negative coefficient of $\frac{6}{7}$, and so on across the girder.

Fig. 11, which for convenience shows the shape of the girder distorted, gives all the positive and negative coefficients for the load at each panel point in turn together with their summations. The chords have maximum coefficients with all panel points loaded and these coefficients are the same as for dead load. Members U0-L0 and U0-L1 also have maximum coefficients with all panels loaded and this is again the same condition as for dead load. Member U1-L1 has maximum positive coefficient with points L2, L3, L4, L5 and L6 loaded, and maximum negative coefficient with only point L1 loaded. Similarly member U1-L2 has maximum negative coefficient with L2, L3, L4, L5 and L6 loaded, and maximum positive coefficient with only L1 loaded. In the same way members U2-L2 and U2-L3 have maximum coefficients of one sign with L3, L4, L5 and L6 loaded, and maximum coefficients of the opposite sign with L1 and L2 loaded, and so on along the girder for each pair of diagonals. This means that for any pair of diagonals maximum load occurs when the longer segment of the girder is loaded, and maximum reversal occurs when the shorter segment is loaded, which is the basis for calculating loads in these members by the influence diagram method.

The coefficients in Figs. 10 and 11 can be written down in much simpler form, and this has been done in Fig. 12, the web member coefficients given on the right-hand half of the girder being the reversals to corresponding members on the left-hand half. The denominator in all these coefficients equals the number of panels in the girder, and the numerators conform to the following general system :—

Numerator	21	15	10	6	3	1	0
First difference	6	5	4	3	2	1	
Second difference	1	1	1	1	1		

This system is of general application and with this in mind the web coefficients can be written down immediately, starting at one end with a coefficient of 0—indicating that there is no reversal in

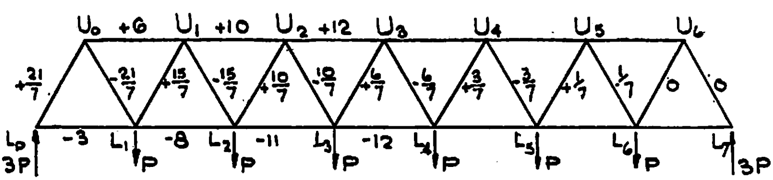


Fig. 12—Method of writing up coefficients for live load.

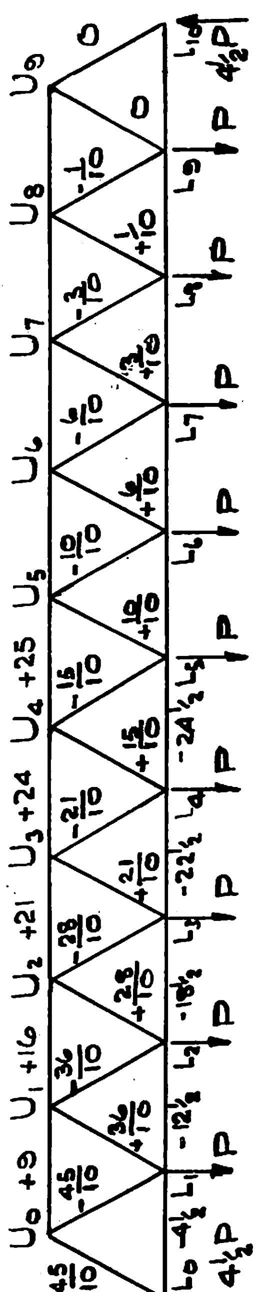


FIG 13 - COEFFICIENTS FOR LIVE LOADS ON WARREN GIRDER WITH 10 BAYS

the end panel—and increasing numerators by differences of 1, 2, 3, 4, 5 and 6.

This general principle holds good irrespective of the number of panels in the girder. For instance in Fig. 13 with 10 panels the deniminator is 10, and the numerators starting from one end are 0, 1, 3, 6, 10, 15, 21, 28, 36 and 45, the maximum coefficient of 45/10 equalling $4\frac{1}{2}$, which is the dead load coefficient for this panel.

For a girder with say 16 panels the denominator would be 16, and the numerators $0+1+2+3+4+5+6+7+8+9+10+11+12+13+14+15$, that is, 0, $^{1}/_{16}$, $^{3}/_{16}$, $^{6}/_{16}$, $^{10}/_{16}$, $^{15}/_{16}$, $^{21}/_{16}$, $^{28}/_{16}$, $^{36}/_{16}$, $^{45}/_{16}$, $^{55}/_{16}$, $^{66}/_{16}$, $^{78}/_{16}$, $^{91}/_{16}$, $^{105}/_{16}$ and $^{120}/_{16}$.

It should be noted that with maximum and minimum coefficients (as they are sometimes called) for the web members, the principle that the chord coefficients add up across the diagonals still holds good with the proviso that for each diagonal both the maximum and minimum coefficients must be included in the addition. For example, in Fig. 13 a section to the right of U1-L1 which corresponds to a section to the left of U8-U9 gives a summation of $-12\frac{1}{2} - ^{38}/_{10} + ^{1}/_{10} + 16 = 0$. It may also be noted that an algebraic summation of maximum and minimum coefficients for any web member will equal the dead load coefficient for the same member. This means that the maximum live load coefficient exceeds the dead load coefficient by the amount of the live load reversal.

It is now possible to take a few examples of different types of girder to show how the general principle of live load coefficients is applied.

Fig. 14 gives live load coefficients for a Pratt truss with an even number of panels. The chord coefficients are as for dead loads, the denominator of the web coefficients is 8, this being the number of panels in the girder, and the numerators follow the system of additions already explained. The web coefficients on the right-hand half of the girder are the reversals for the corresponding members in the left-hand half. Members U1-L1 and U7-L7 are hangers transferring the load to the upper chord and therefore have a coefficient of -1.

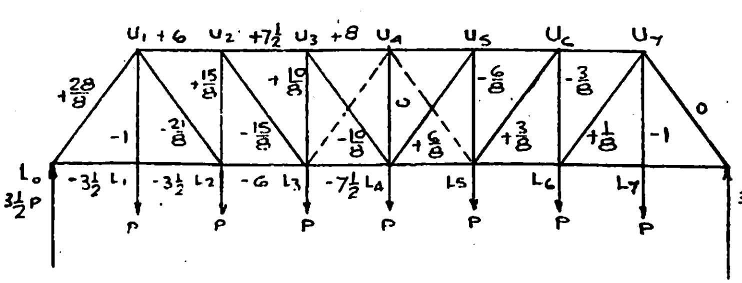


Fig. 14—Coefficients for live load for Pratt truss with 8 Panels.

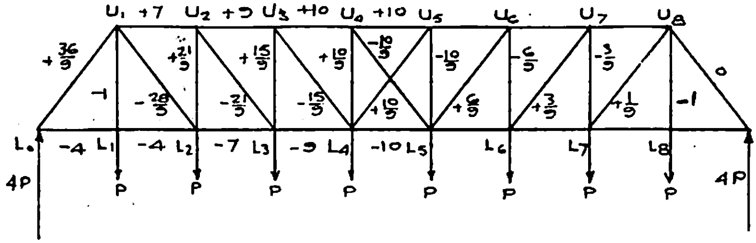
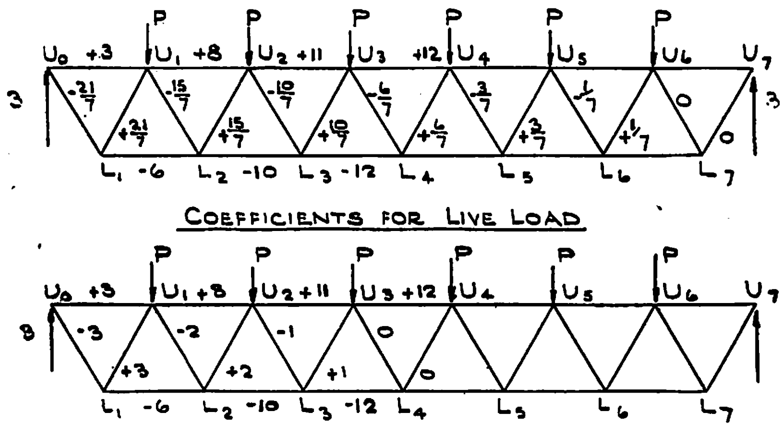


Fig. 15—Coefficients for live loads for Pratt truss with 9 Panels.

Fig. 15 gives coefficients for a similar girder with an odd number of panels and the chief point to note is the sign of the coefficients in the centre panel. If it is assumed that members U5-L5 and U5-L4 are effective with members U4-L5 and U4-L4 ineffective, the coefficients for U5-L5 and U5-L4 are $-{}^{10}/_{9}$ and $+{}^{10}/_{9}$ respectively. Alternatively if the reverse assumption is made the coefficients for U4-L5 and U4-L4 are $-{}^{10}/_{9}$ and $+{}^{10}/_{9}$ respectively. This means that all four members in the centre panel will be in tension or compression according to how the loads are assumed to act, and since compression will give the more severe design condition they are all designed as struts with loads based upon the positive coefficients, and then checked if necessary for tension.

Fig. 16 gives dead load and live load coefficients for a Warren girder deck span. The dead load coefficients are written down first, with those for live load following, and these should be clear after previous explanations. In this example, and in others



COEFFICIENTS FOR LIVE LOAD

COEFFICIENTS FOR DEAD LOAD.

FIG-16 COEFFICIENTS FOR WARREN GIRDER DECK SPAN

following, separate diagrams have been drawn for dead load and live load, but the coefficients for both can be written on the same diagram using different coloured pencils for clarity.

Fig. 17 gives dead load and live load coefficients for a Warren girder through span with verticals. The function of the verticals is to transfer the load at L1, L3, etc., to the upper chord and their coefficients will therefore be -1 for both dead and live loads. The girder has 14 panels so that the web member live load coefficients have a denominator of 14 with numerators adding up on the same principle as before, while chord coefficients again add up across the diagonals (ignoring the verticals).

Fig. 18 gives dead load and live load coefficients for a Warren girder deck span with verticals, the procedure being generally as for Fig. 17.

Fig. 19 gives the procedure for finding maximum and minimum coefficients for a Pratt truss with sub-divided panels. Dead load coefficients for this truss have already been given in Fig. 5, and for purposes of comparison with the live load figures the dead load coefficients for the main web members have been shown encircled in Fig. 19. The sub-division of the bracing system introduces a complication in dealing with the main web members, but subject to this variation the general principle remains unchanged.

For live loads as for dead loads the sub-verticals will all have a coefficient of -1, since their function is to transfer load to the centre intersection points, at which points these loads actually enter the bracing system. In view of this there can be no reversals in these members.

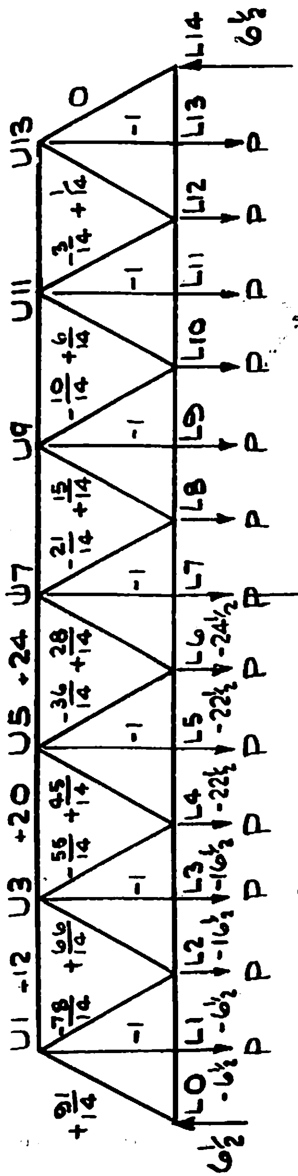
Similarly all the sub-diagonals will have a constant coefficient of $+\frac{1}{2}$, since half the load from each sub-vertical passes into these members, the other half passing into the upper portion of the main diagonals and in the case of C1-L2 into the lower portion of the end post. The sub-diagonals therefore also can have no reversal, since they transfer load into the main system.

Main verticals U2-L2 and U14-L14 are also hangers and, as in the case of dead load, will have a coefficient of -2 to transfer the loads at L2 and L14 and half of the loads from each adjacent sub-vertical.

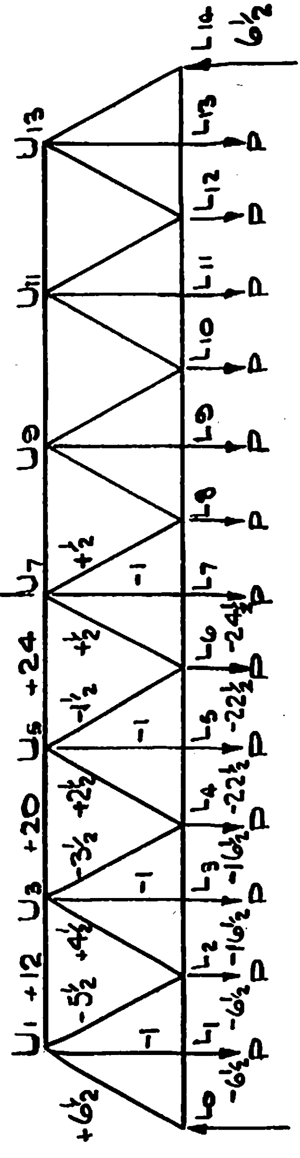
The live load coefficients for the chords will be the same as for dead load and can be copied from Fig. 5.

This leaves only the coefficients for the main web members and the easiest way to tackle these at this stage is to consider first the effect of the loads at L2, L4, L6, L8, L10, L12 and L14 on the basis of previous procedure, and then the additional effect of the loads from the sub-verticals.

For the first set of loads at L2, L4, etc., there are eight panels and the denominator will therefore be 8, the numerators having



COEFFICIENTS FOR LIVE LOADS



COEFFICIENT FOR DEAD LOADS

FIG. 17 COEFFICIENTS FOR WARREN GIRDER THROUGH SPAN.
WITH VERTICAL MEMBERS.

differences of 1, 2, 3, 4, 5, 6 and 7. The upper set of coefficients on the main web members cover this condition and their values are therefore $\frac{1}{8}$, $\frac{3}{8}$, $^6/_8$, $^{10}/_8$, $^{15}/_8$, $^{21}/_8$ and $^{28}/_8$.

Taking next the loads hanging from the centre intersection points C1 to C15, half of the load at C1 and C3 passes to L2 through C1-L2 and C3-L2, giving in effect a total load of 2P at L2. The other half of the load at C1 passes directly into C1-L0 and the second half of the load at C3 passes through C3-U2 into the end post. So far as the end post is concerned the loads at C1, L2 and C3 can therefore be replaced by loads of $\frac{1}{2}$P at C1, 2P at L2, and $\frac{1}{2}$P at U2. Similarly at the other end of the girder, so far as the end post is concerned, the loads at C13, L14 and C15 can be replaced by loads of $\frac{1}{2}$P at U14, 2P at L14, and $\frac{1}{2}$P at C15.

In the same way half of the load at C5 passes to L4 through C5-L4, and the other half passes through C5-U4 and U4-L4 to L4, and so far as members L4-U2 and U2-L0 are concerned the load at C5 may be assumed to act at L4.

Similarly, so far as all main web members to the left of L6 are concerned, the load at C7 may be assumed to act at L6.

On this basis a new loading system (for the web members) may be assumed consisting of 2P at L2, L4, L6, L10, L12 and L14, $\frac{1}{2}$P at C1, U2, U14 and C15, and P at L8.

Taking first the additional loads at L2, L4, L6, L10, L12 and L14, this is a repetition of the first set of loads except that there is no additional load at L8. This means that in the system of numerator differences (1, 2, 3, 4, 5, 6 and 7) the difference figure of 4 for panel L8-L6 is omitted, since this difference would have been produced by a load at L8, which is non-existent. Hence for members L8-U6 and U6-L6 the coefficients are the same as for L10-U10 and U10-L8, and the next difference of 5 is added to L6-U4 and U4-L4.

The second set of coefficients given for the main web members, covering the additional loads at L2, L4, L6, L10, L12 and L14, are therefore $\frac{1}{8}$, $\frac{3}{8}$, $^6/_8$, $^6/_8$, $^{11}/_8$, $^{17}/_8$ and $^{24}/_8$.

The half loads at C3, C5 and C7 will produce additional coefficients of $-^4/_8$ in C3-U2, C5-U4, and C7-U6, and these will automatically induce an additional coefficient of $+^4/_8$ in U6-L6, U4-L4, U2-C1, and C1-L0, while the half load at C1 will give a further additional coefficient of $+^4/_8$ in C1-L0. There will be no additional reversal effect from these half loads since the reversals have already been accounted for in writing up for the additional loads at the main verticals which include these half loads.

This completes the analysis and a check on the total coefficients for the main web members may be obtained by an algebraic summation of the maximum and minimum coefficients for each

member, which should equal the dead load coefficient, while the coefficient for the lower portion of the end post will equal the shear in the end panel.

This detailed analysis is necessarily rather lengthy, but an examination of the results will show that the coefficients again follow a well-defined principle. Considering first the main verticals and main diagonals on the right-hand half of the girder which give the values of the reversals, the differences in the numerators are 2, 4 and 6, instead of 1, 2 and 3 in previous examples. That is to say, the introduction of sub-verticals doubles the differences in the coefficients. Considering next the main verticals and the upper half of the main diagonals on the left-hand half of the girder, which give the values of the maximum coefficients, the same system of doubled differences continues right along the girder. Thus from right to left (taking the upper half only of the main diagonals on the left-hand half) the differences are 2, 4, 6, 8, 10, 12 and 14. The lower portions of diagonals U2-L4, U4-L6, and U6-L8 will have a coefficient $^4/_8$ less than the upper portion, and the lower portion of the end post will have a coefficient $^4/_8$ greater than the upper portion, this difference being produced by the effect of the half load at the centre intersection points.

Fig. 20 gives the procedure for finding maximum and minimum coefficients for a Warren girder with sub-divided panels. Dead load coefficients have already been given in Fig. 6, and the live load coefficients follow the same general principle as for the Pratt truss in Fig. 19 with minor variations to suit the shape of the girder.

It is necessary to understand how the sub-vertical loads enter the main bracing system since this gives the key to the main diagonal coefficients. The load at L7 (actually acting at C7) is shared between C7-U6 and C7-L6, the half in C7-L6 passing through L6-U6 into the main web system. Similarly the load at C3 is shared between C3-U2 and C3-L2, the half in C3-L2 passing through L2-U2 into the end post. The load at C5 is shared between C5-L4 and C5-L6, the half in C5-L6 passing through L6-U6 into the main web system. Similarly the load at C1 is shared between C1-L0 and C1-L2, the half in C1-L2 passing through L2-U2 into the end post.

Taking now the upper portions of the main diagonals, the differences in the numerators will again be 2, 4, 6, 8, 10, 12 and 14 as in the previous example, giving total coefficients (starting from the right) of 0, $^2/_8$, $^6/_8$, $^{12}/_8$, $^{20}/_8$, $^{30}/_8$, $^{42}/_8$ and $^{56}/_8$. Owing to the manner in which the sub-vertical loads are transmitted the lower portions of diagonals U2-L4 and U6-L8 will each have a coefficient $^4/_8$ less than the upper portion, while the lower portions of diagonals U2-L0 and U6-L4 will each have a coefficient $^4/_8$ greater than the upper portion.

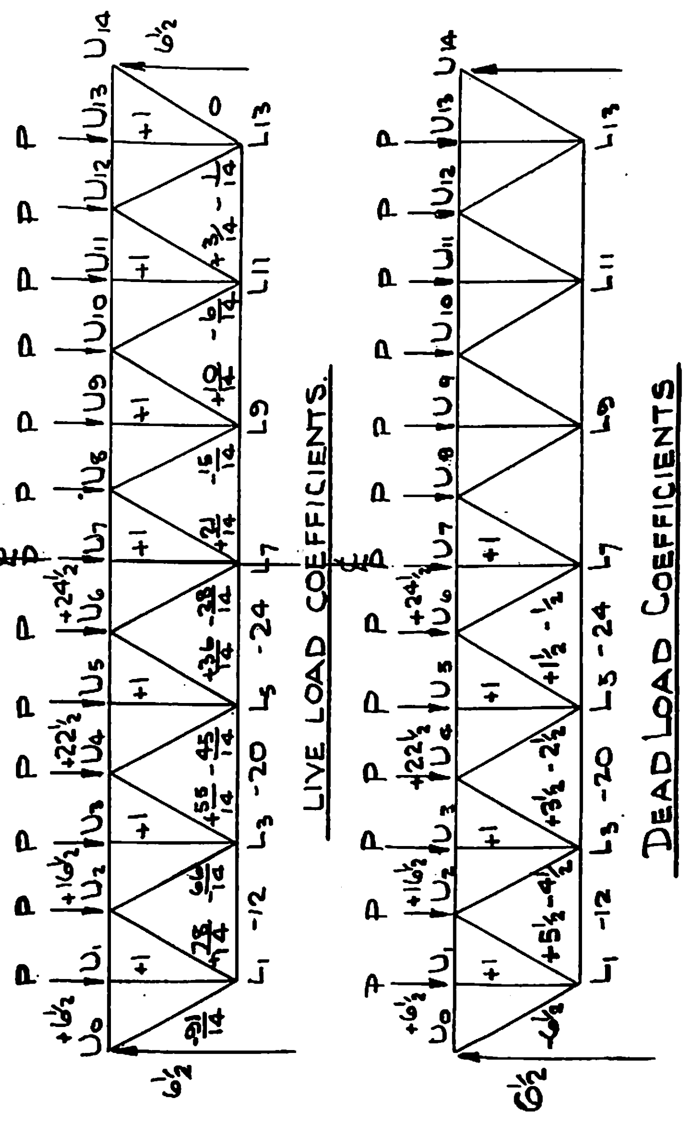


FIG-18 COEFFICIENTS FOR WARREN GIRDER DECK SPAN WITH VERTICALS.

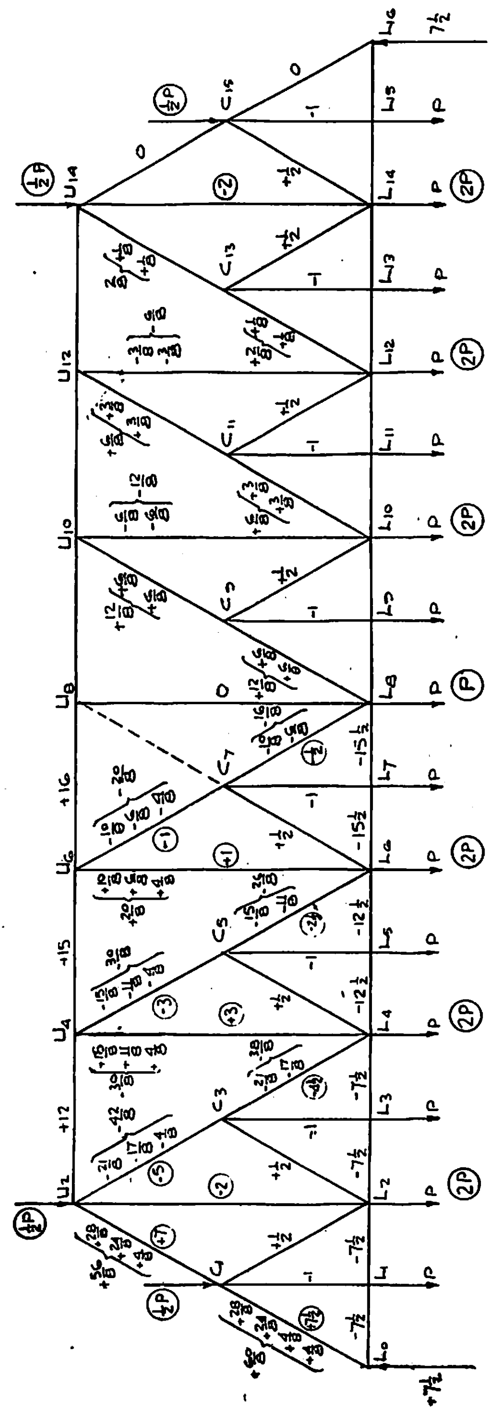


FIG. 19. MAXIMUM & MINIMUM COEFFICIENTS FOR PRATT TRUSS WITH SUB-DIVIDED BAYS.

The detailed coefficients for the main diagonals can, if necessary, be checked following a similar procedure to that for Fig. 19, but with the explanations already given it should now be possible to write up the total coefficients at one step.

Fig. 21 gives dead load and live load coefficients for a K-truss. Owing to the fact that the compression members are of comparatively short length on the weaker axis this type of triangulation gives reasonably economical sections for long span bridges and deserves to be used more widely than at present, since one of the principal aims of good designing is to employ short rather than long compression members.

Procedure is generally similar to that for previous examples but the shape of the truss with two diagonals in each panel involves special consideration to certain points.

Taking first the dead load coefficients it will be clear that the shear in each panel is shared equally by the two diagonals in the panel, and working from mid-span outward towards line U2-L2, the diagonal coefficients will be $\frac{1}{4}$, $\frac{3}{4}$, $1\frac{1}{4}$ and $1\frac{3}{4}$ in the respective panels. In accordance with general lattice girder principles the diagonals sloping downwards towards the centre of gravity of the loading system are in tension and those sloping in the opposite direction are in compression. This means that the lower diagonals in these four panels have negative coefficients and the upper diagonals have positive coefficients.

Vertical members U5-C5, U4-C4 and U3-C3 must equilibrate the vertical components of the upper diagonals U5-C4, U4-C3 and U3-C2, and will therefore have coefficients of $-\frac{3}{4}$, $-1\frac{1}{4}$, and $-1\frac{3}{4}$ respectively. Vertical members C5-L5, C4-L4 and C3-L3 must equilibrate the vertical components of C4-L5, C3-L4 and C2-L3, together with the loads hanging at L5, L4 and L3, and hence C5-L5 must have a coefficient of $-\frac{1}{4}$, C4-L4 a coefficient of $+\frac{1}{4}$, and C3-L3 a coefficient of $+\frac{3}{4}$. Vertical U6-L6 must equilibrate the vertical components of C5-L6 and C7-L6 and the load hanging at L6, which gives this member a coefficient of $-\frac{1}{2}$, which is also the equilibrant of the vertical components of diagonals C5-U6 and C7-U6.

At the end of the girder sub-diagonal C1-L2 has a coefficient of $+\frac{1}{2}$ since it takes half of the load at L1. Hence vertical C2-L2 must have a coefficient of $-1\frac{1}{2}$ to equilibrate the vertical component of C1-L2 and the load hanging at L2.

For equilibrium at C2, vertical C2-U2 must equilibrate C2-L2, and the vertical components of C2-U3 and C2-L3, giving a coefficient for this member of -5.

End post U2-C1 must have a coefficient of $+5$ so that its vertical component may equilibrate U2-C2, and end post C1-L0 will have

a coefficient of $+5\frac{1}{2}$ to account for half of the load from L1 and to equal the shear in the end panel.

This completes all the web members, and if a check is made at any of the central intersection points it will be found that the coefficients cancel out. For example, at C3 there is an upward force corresponding to a total coefficient of $2\frac{1}{2}$ from the verticals which is negatived by an equal downward force corresponding to a total coefficient of $2\frac{1}{2}$ for the diagonals. Also the horizontal components of diagonals C3-U4 and C3-L4 cancel out.

For the upper chord U2-U3 must have the same coefficient as end post U2-C1 for equilibrium, and adding up across the upper diagonals U3-U4 has a coefficient of $+5+1\frac{3}{4} = +6\frac{3}{4}$, U4-U5 has a coefficient of $+6\frac{3}{4}+1\frac{1}{4} = +8$, and U5-U6 has a coefficient of $+8+\frac{3}{4} = +8\frac{3}{4}$.

For the lower chords L0-L2 has a coefficient of $-5\frac{1}{2}$ to agree with C1-L0 and the reaction, and L2-L3 has a coefficient of -5 to agree with C1-U2 and to equilibrate L0-L2 and the horizontal component of C1-L2. L2-L3 must also have the same coefficient (but of opposite sign) as U2-U3 so that a section cut through these two members and through diagonals C2-U3 and C2-L3 will give an algebraic summation of zero. Adding up across the lower diagonals L3-L4 has a coefficient of $-(5+1\frac{3}{4}) = -6\frac{3}{4}$, L4-L5 has a coefficient of $-(6\frac{3}{4}+1\frac{1}{4}) = -8$, and L5-L6 has a coefficient of $-(8+\frac{3}{4}) = -8\frac{3}{4}$. In the same manner as L2-L3 and U2-U3, all these lower chord members have the same numerical coefficients as the upper chord members in the same panels, in order to give algebraic summations of zero in each panel. This completes the dead load analysis, and although the detailed explanation is necessarily rather lengthy an examination of the results will show that these coefficients can be written down quickly once the general principle is understood.

Turning now to the live loads the chords will again have the same coefficients as for dead loads. The end posts will have no reversals and will therefore have the same coefficients as for dead loads, and with 12 panels in the girder giving a denominator of 12 the live load coefficients for L0-C1 and C1-U2 may be written as $+{}^{66}/_{12}$ and $+{}^{60}/_{12}$ respectively.

For the diagonals the differences in the numerators will follow the general system of 1, 2, 3, 4, 5, 6, 7, 8 and 9, but as there is no reversal in the second panel which is part of the end post the minimum coefficient of ${}^{1}/_{12}$ in this panel is omitted and the combined diagonal coefficients starting from the third panel on the right are ${}^{3}/_{12}$, ${}^{6}/_{12}$, ${}^{10}/_{12}$, ${}^{15}/_{12}$, ${}^{21}/_{12}$, ${}^{28}/_{12}$, ${}^{36}/_{12}$ and ${}^{45}/_{12}$. These will be shared equally between the upper and lower diagonals and this gives the numerical value of all diagonal coefficients. With regard to the signs, diagonals to the left of mid-span must have the same

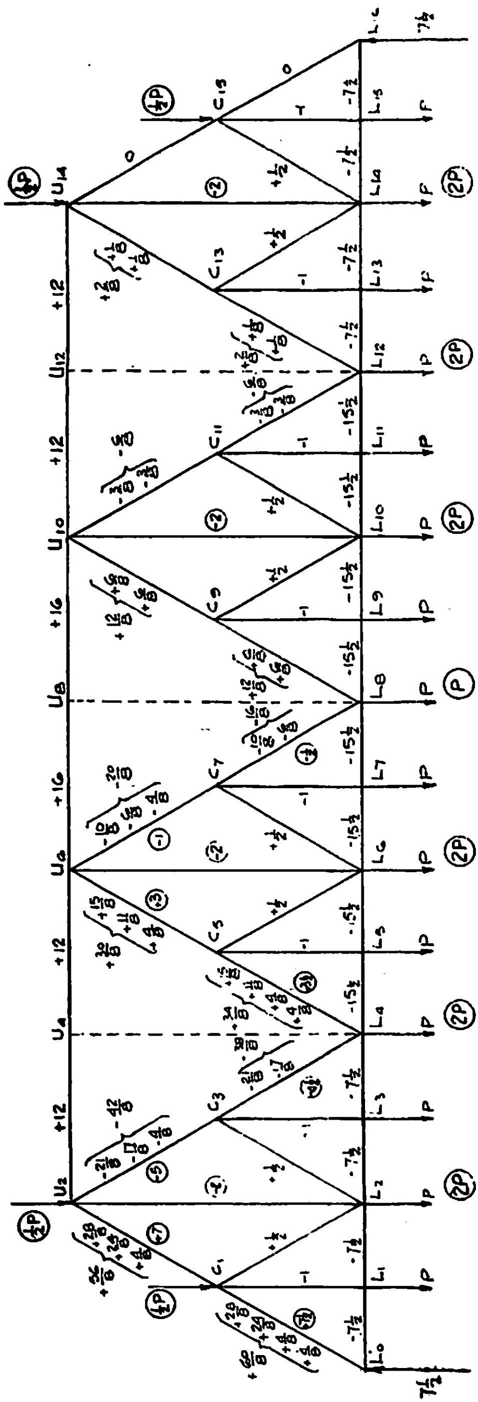


FIG. 20 - MAXIMUM & MINIMUM COEFFICIENTS FOR WARREN GIRDER WITH SUB-DIVIDED BAYS

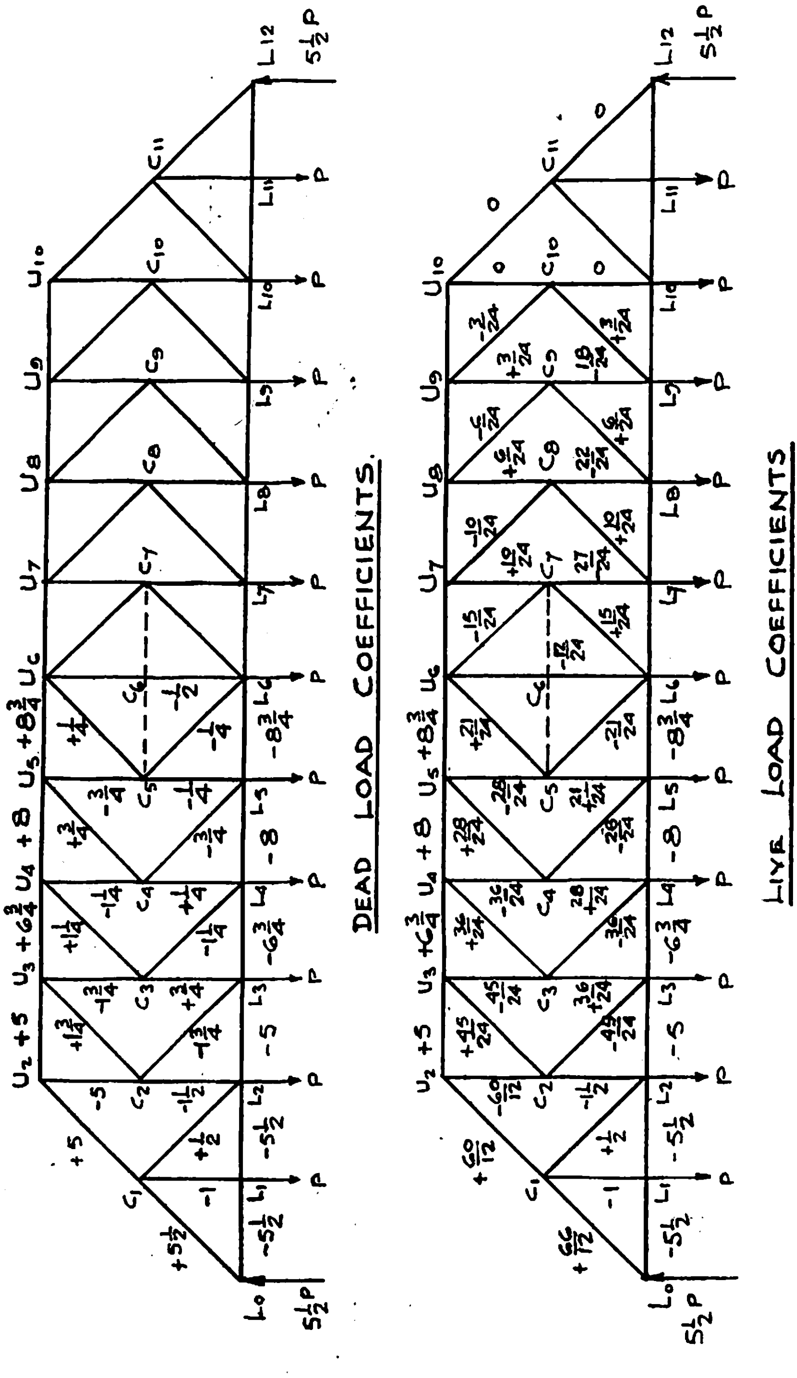


FIG. 21 - DEAD LOAD & LIVE LOAD COEFFICIENTS FOR K-TRUSS.

sign as for dead loads, and therefore those to the right giving the reversals will be of opposite sign on corresponding members. It will be found that an algebraic summation of maximum and minimum coefficients for any diagonal will equal the dead load coefficient.

The upper verticals, as in the case of dead loads, must equilibrate the vertical components of the adjacent diagonals and this will fix their coefficients. For example, U2-C2 must have the same coefficient as U2-C1, U3-C3 the same coefficient as U3-C2, and so on.

Lower vertical C2-L2 is actually a hanger without a reversal and will have the same coefficient as for dead load. This may be checked by making a summation at C2, including in the summation the reversals for C2-U3 and C2-L3 written against members C10-U9 and C10-L9.

Vertical C3-L3 will have a maximum coefficient for the same loading condition which gives maximum coefficients for diagonals C3-L4 and C3-U4, that is with loads at panel points L4 to L11, with points L1, L2, L3 unloaded. This means that with loads at L4 to L11, C3-L3 must equilibrate the vertical component of C3-L4 giving a maximum coefficient of $+{}^{36}/_{24}$. The minimum coefficient for this member will equal the difference between the maximum coefficient and the dead load coefficient of $+\frac{3}{4}$, that is $-{}^{18}/_{24}$, written against corresponding member C9-L9, and represents the condition with loads at L9, L10 and L11, with panel points L1 to L8 unloaded.

In the same way vertical C4-L4 will have maximum coefficient with points L5 to L11 loaded and points L1, L2, L3 and L4 unloaded. Its maximum coefficient will be the vertical component of diagonal C4-L5, that is $+{}^{28}/_{24}$. The minimum coefficient, written against corresponding member C8-L8, will again be the difference between the maximum coefficient and the dead load coefficient of $+\frac{1}{4}$, that is $-{}^{22}/_{24}$, and represents the condition with loads at L8, L9, L10 and L11, with panel points L1 to L7 unloaded.

Similarly vertical C5-L5 will have maximum coefficient with panel points L6 to L11 loaded and points L1 to L5 unloaded. The maximum coefficient will be the vertical component of diagonal C5-L6, that is $+{}^{21}/_{24}$. The minimum coefficient, written against C7-L7, is the difference between the maximum coefficient and the dead load coefficient, that is $-{}^{27}/_{24}$.

Centre vertical L6-U6 is merely a hanger, without reversal, having the same coefficient as for dead loads. This can be checked by loading points L6 to L11 with points L1 to L5 unloaded, which is the condition for maximum coefficient of $-{}^{21}/_{24}$ in C5-L6. For this loading condition the coefficient for C7-L6 (not a reversal)

would be $+{}^{9}/_{24}$, and vertical L6-U6 equilibrates diagonals C5-L6 and C7-L6 and the load at L6. The reversal for C5-L6, written against C7-L6 is obtained with points L7 to L11 loaded and points L1 to L6 unloaded. For this loading condition both C5-L6 and C7-L6 have the same coefficient of ${}^{15}/_{24}$ but of opposite signs, so that the vertical component of these diagonals for this condition cancel out leaving no reversal load in L6-U6.

This detailed analysis for the live loads, like that for dead loads, appears to be rather lengthy, but the actual procedure is straightforward and logical in accordance with the principles previously explained and there should be no difficulty after a little practice.

In the case of light footbridges it sometimes happens that the loads are applied at alternate panel points only. Examples of

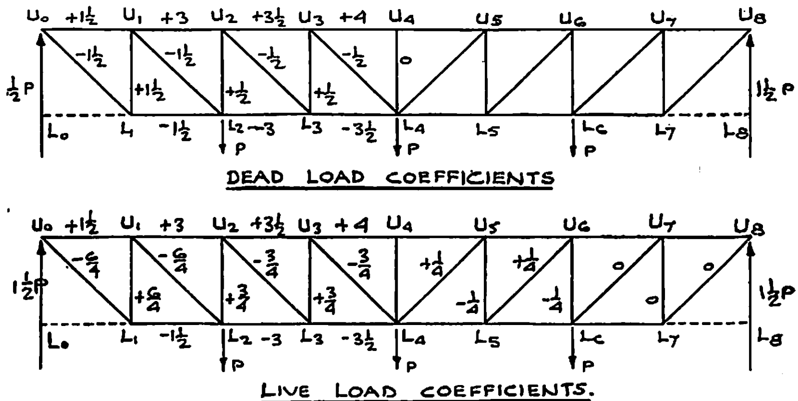


FIG. 22 - COEFFICIENTS FOR LATTICE GIRDER LOADED AT ALTERNATE POINTS ONLY

this are shown in Figs. 22 and 23 which give the procedure for a light Pratt truss and Warren girder respectively.

In such cases with only light loading all the loads are sometimes assumed to act as dead loads which would be correct for the chords and end panels of web members, but is not correct for the inner web members although the error is not very great.

The procedure for writing down the coefficients should be self-explanatory, the chief points to note being that although the girders have eight panels there are only four panels of loading, so that the denominators for live load coefficients for the web members will be four and not eight, and the differences of 1, 2 and 3 for the numerators are also applicable to the load panels.

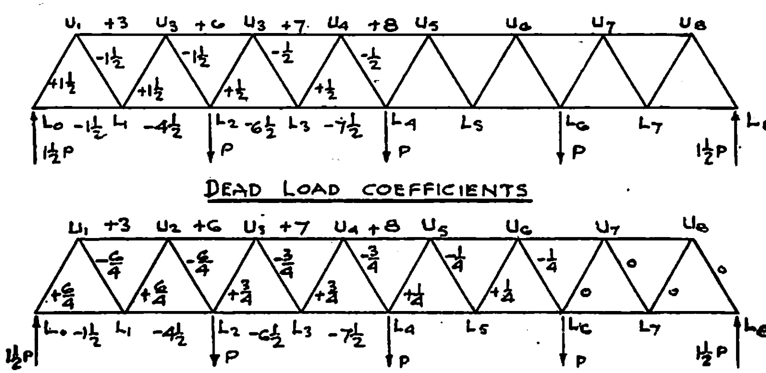


FIG. 23. COEFFICIENTS FOR WARREN GIRDER LOADED AT ALTERNATE PANEL POINTS ONLY.

BASIS OF PROCEDURE FOR EQUIVALENT UNIFORMLY DISTRIBUTED LIVE LOADS (E.U.D.L.L.).

B.S. 153, Part 3A, 1954, for Girder Bridges, gives particulars of live loads for railway bridges and for highway bridges normally applicable to conditions in Great Britain.

For railway bridges Type R.A.1 Loading covers normal construction of 4′-8½″ gauge, and for this loading tables of equivalent uniformly distributed loads for bending moment and for shear are included in the specification which are invariably used for main girder design to avoid the tedious calculations required for concentrated loads. These tables do not include impact effect and an alternative loading condition—Type R.B. Loading—is given setting out equivalent uniform loads for various impact requirements.

For highway bridges Type H.A. Loading covers normal requirements. This loading has one heavy axle transmitting about double the load of the other axles and instructions are given in the specification for reducing this loading to an equivalent uniform load plus a single knife edge load to account for the extra load from the heavy axle.

Ministry of Transport Standard Loading for Highway Bridges is slightly different to the loading given in B.S. 153, but the general principle of a single heavy axle is maintained and this loading is also reduced to an equivalent uniform load plus a single knife edge load.

For bridges supporting either railways or highways it is therefore possible and usually much more convenient to design the main girders to take the equivalent uniform load, bearing in mind that for highway bridges an additional single moving load must also be accounted for.

These requirements can conveniently be dealt with by the method of shear coefficients, which reduces the labour of calculation to a minimum. For a uniform load the coefficients for the chord members will be the same as for dead loads since these members carry maximum load with the bridge fully loaded, which is the dead load condition, so that the only explanation needed is the procedure for the web members. For a single moving load the coefficients for both the web members and the chords need explanation.

Taking first the procedure for uniform loads, this is based directly upon the influence lines for the web members and upon an understanding of the method of calculating the areas of the influence diagrams.

Consider Fig. 24 which gives the influence lines for shear in the various panels. Maximum positive shear in panel L0-L1

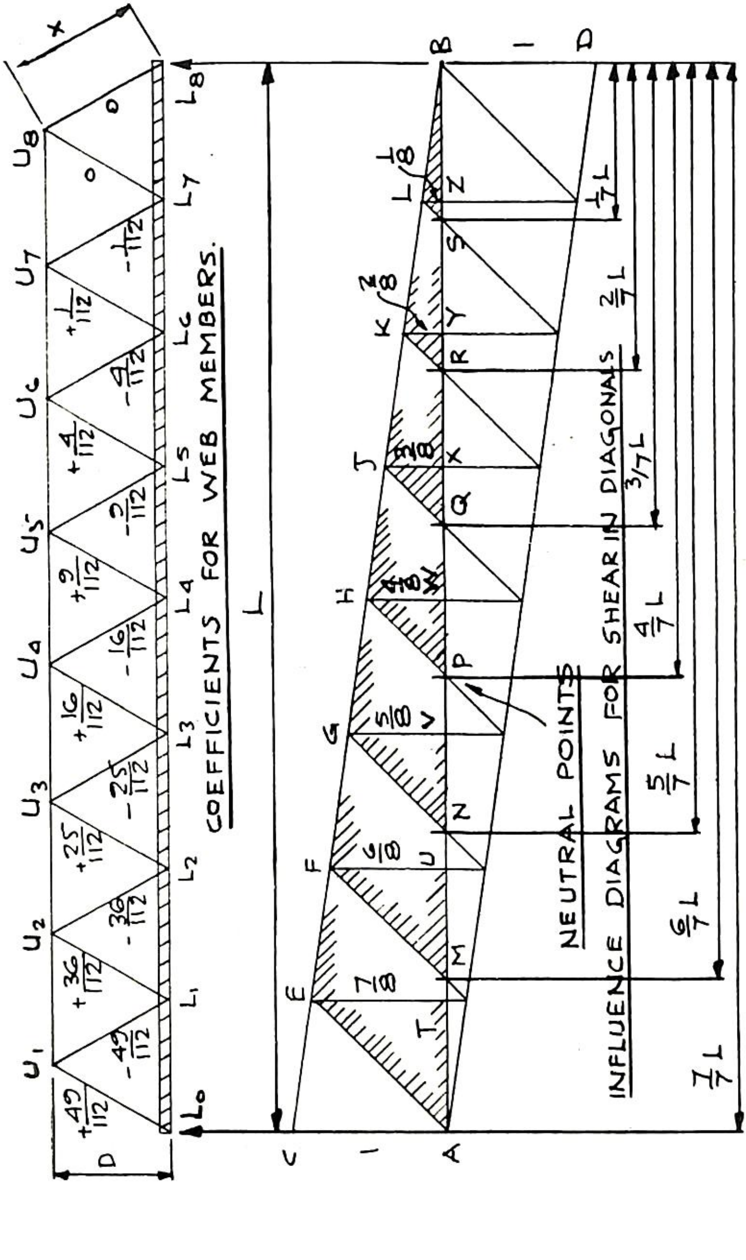


FIG. 24 WEB COEFFICIENTS FOR WARREN GIRDER WITH U.D.L.L.

occurs with the span fully loaded and equals the area of triangle AEB multiplied by the load for foot run. Maximum positive shear in panel L1-L2 occurs with the portion of span from reaction B to neutral point M loaded and equals the area of triangle MFB multiplied by the load per foot run. Maximum negative shear in panel L1-L2 occurs with the portion of span from reaction A to neutral point M loaded. Length AM equals length SB at the right-hand end, so that maximum negative shear in panel L1-L2 equals the area of triangle SLB multiplied by the load per foot run.

In the same way maximum positive shear in panel L2-L3 equals the area of triangle NGB multiplied by the load per foot run and the maximum negative shear in this panel equals the area of triangle RKB multiplied by the load per foot run. Maximum positive shear in panel L3-L4 equals the area of triangle PHB multiplied by the load per foot run, and maximum negative shear in this panel equals the area of triangle QJB multiplied by the load per foot run.

In calculating the areas of the above triangles it may be proved that loaded length SB equals one-seventh of the span (for eight panels), loaded length RB equals two-sevenths of the span, and so on, as shown in Fig. 24, while the heights of the respective triangles (for eight panels) are $\frac{1}{8}$, $^2/_8$, $\frac{3}{8}$, $^4/_8$, $\frac{5}{8}$, $^6/_8$ and $\frac{7}{8}$.

Hence area	of	triangle	SLB	=	$\frac{1}{2} \times \frac{1}{8} \times {}^1/_7 \times$ Span.
,,	,,	,,	RKB	=	$\frac{1}{2} \times {}^2/_8 \times {}^2/_7 \times$ Span.
,,	,,	,,	QJB	=	$\frac{1}{2} \times \frac{3}{8} \times {}^3/_7 \times$ Span.
,,	,,	,,	PHB	=	$\frac{1}{2} \times {}^4/_8 \times {}^4/_7 \times$ Span.
,,	,,	,,	NGB	=	$\frac{1}{2} \times \frac{5}{8} \times {}^5/_7 \times$ Span.
,,	,,	,,	MFB	=	$\frac{1}{2} \times {}^6/_8 \times {}^6/_7 \times$ Span.
,,	,,	,,	AEB	=	$\frac{1}{2} \times \frac{7}{8} \times {}^7/_7 \times$ Span.

From the above it will be clear that both the denominators and the numerators follow a fixed rule.

If number of panels in girder $= n$

Then denominator $= 2 \times n \times (n-1)$.

Numerators starting from the right-hand panel, which has no reversal, are the squares of 0, 1, 2, 3, 4, 5, 6 and 7.

The length coefficient, as before, will be X/D, and for a load per foot run of p tons the load in the end panel will be

$$\frac{49}{112} p\,L \times \frac{X}{D}.$$

The value p L can conveniently be replaced by W, the total distributed load on the girder, and the load in the end panel will then read $\frac{49}{112} W \times \frac{X}{D}$.

The expression $\frac{WX}{112D}$ is common to all panels and for each panel in turn the maximum load and the reversal in the members can be found by multiplying this expression by the numerator.

If the value of $\frac{WX}{112D}$ be called K, then the loads in the members are as follows :—

Load in L0-U1	=	+49K
,, ,, U1-L2	=	−49K
Maximum load in L1-U2	=	+36K
Reversal ,,	=	− 1K
Maximum load in U2-L2	=	−36K
Reversal ,, ,,	=	+ 1K
Maximum load in L2-U3	=	+25K
Reversal ,, ,,	=	− 4K
Maximum load in U3-L3	=	−25K
Reversal ,, ,,	=	+ 4K
Maximum load in L3-U3	=	+16K
Reversal ,, ,,	=	− 9K
Maximum load in U3-L3	=	−16K
Reversal ,, ,,	=	+ 9K

Thus in practice all that is required is to find first the value of the expression $\frac{WX}{112D}$, and for each panel in turn multiply this by 1^2, 2^2, 3^2, etc., bearing in mind that the coefficients given for the right-hand half of the girder represent the reversals for the corresponding members in the left half.

In the end panel, since there is no reversal, the same result would have been obtained by using the dead load coefficient of $3\frac{1}{2}$ and assuming a concentrated load of $\frac{pL}{8}$ at the panel points. The same result would also have been obtained in the end panel by using the live load coefficient of $^{28}/_{8}$ for concentrated live loads, but in the remaining panels a distributed live load will give slightly smaller loads in the web members than would be obtained for loads concentrated at the panel points.

The general principle for finding the value of the denominator and numerators explained above holds good irrespective of the number of panels in the girder. For example, with 10 panels the denominator would be $2 \times 10 \times 9 = 180$, and the numerators starting from the right would be 0^2, 1^2, 2^2, - - - - - 9^2.

Consider next the case of the single moving knife edge load which has to be taken into account in highway bridge design.

This load will occur at each panel point in turn and since the effect on all members will vary according to the position of the load what is required is the maximum effect on each member.

This condition has already been discussed in detail in connection with Figs. 10 (*a*)—(*f*), and although in that example the purpose was to find the cumulative effect of moving loads when all panel points are loaded the same general principle holds good when considering only a single load acting at each point in turn.

For the Warren girder in Fig. 24 the coefficients for a single moving load have been listed separately for convenience in Fig. 25. Following the procedure of Figs. 10 (*a*)-(*f*) the maximum coefficients for members L0-U1 and U1-L1 will be $+\frac{7}{8}$ and $-\frac{7}{8}$ respectively with the load located at L1. This condition will also give maximum coefficients for L0-L1 and U1-U2, which will be $-\frac{7}{8}$ and $+{}^{14}/_{8}$ respectively, the latter being the addition of the coefficients for diagonals L0-U1 and U1-L1, or in other words $2 \times \frac{7}{8}$.

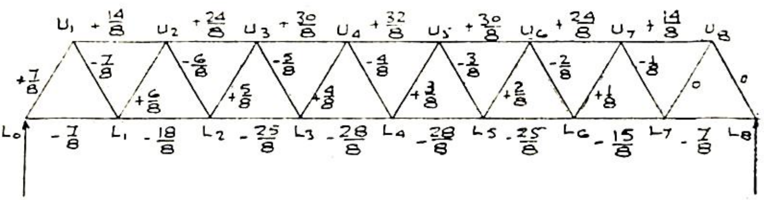


Fig. 25—Coefficients for single moving load, crossing the girder from left to right.

With the load at L2 all the web members to the left of the load will have a coefficient of $\frac{6}{8}$, but since this is less than the maximum coefficients for L0-U1 and U1-L1 already found it can be ignored for these two members. This position of the load however will give maximum coefficients of $+\frac{6}{8}$ and $-\frac{6}{8}$ for diagonals L1-U2 and U2-L2 respectively, and will also give maximum coefficients for chords L1-L2 and U2-U3. L1-L2 must have a maximum coefficient of $-{}^{18}/_{8}$ since with the load at L2 all web members to the left of L2 have a coefficient of $\frac{6}{8}$ and the coefficient for L1-L2 will therefore be the addition of these for diagonals L0-U1, U1-L1, and L1-U2, in other words $3 \times \frac{6}{8}$. Similarly the coefficient for U2-U3 will be the addition of coefficients for diagonals L0-U1, U1-L1, L1-U2, and U2-L2, in other words $4 \times \frac{6}{8}$.

In the same way with the load at L3, all web members to the left of L3 have a coefficient of $\frac{5}{8}$, so that the coefficient for L2-L3 will be $-(5 \times \frac{5}{8}) = -{}^{25}/_{8}$, and that for U3-U4 will be $+(6 \times \frac{5}{8} = +{}^{30}/_{8}$. This position of the load gives maximum coefficients of $+\frac{5}{8}$ and $-\frac{5}{8}$ for diagonals L2-U3 and U3-L3 respectively.

Similarly with the load at L4, all web members to the left of L4 have a coefficient of $\frac{4}{8}$, so that the coefficient for L3-L4 will be $-(7 \times \frac{4}{8}) = -{}^{28}/_{8}$, and that for U4-U5 will be $+(8 \times \frac{4}{8}) = +{}^{32}/_{8}$. This position of the load gives maximum coefficients of $+\frac{4}{8}$ and $-\frac{4}{8}$ for diagonals L3-U4 and U4-L4 respectively.

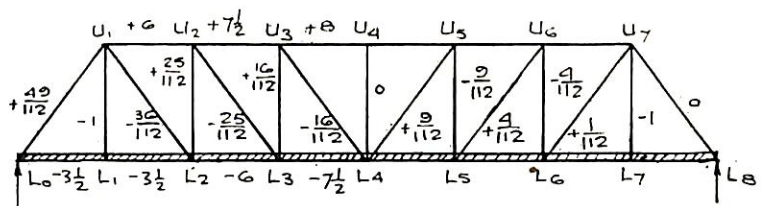
This gives all the maximum coefficients for the diagonals and the chords, and the reversals for the diagonals are found by continuing the process and placing the load at L5, L6 and L7 in turn.

The general result is that denominators for all coefficients equal the number of panels in the girder, and the numerators for the diagonals starting from the right are 0, 1, 2, 3, 4, 5, 6 and 7.

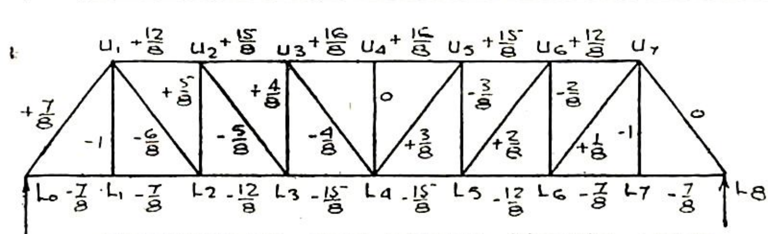
For the chords the numerators are :—

L0-L1	=	1×7	=	7
U1-U2	=	2×7	=	14
L1-L2	=	3×6	=	18
U2-U3	=	4×6	=	24
L2-L3	=	5×5	=	25
U3-U4	=	6×5	=	30
L3-L4	=	7×4	=	28
U4-U5	=	8×4	=	32

This again is a universal system irrespective of the number of panels in the girder. Thus with 10 panels the common denominator would be 10, the diagonal coefficients starting from the right would be 0, ${}^{1}/_{10}$, ${}^{2}/_{10}$ - - - - - ${}^{9}/_{10}$, the lower chord coefficients starting



COEFFICIENTS FOR UNIFORMLY DISTRIBUTED LIVE LOAD



COEFFICIENTS FOR SINGLE MOVING LOAD.

FIG. 26 - COEFFICIENTS FOR U.D.L.L. & SINGLE MOVING LOAD FOR PRATT TRUSS

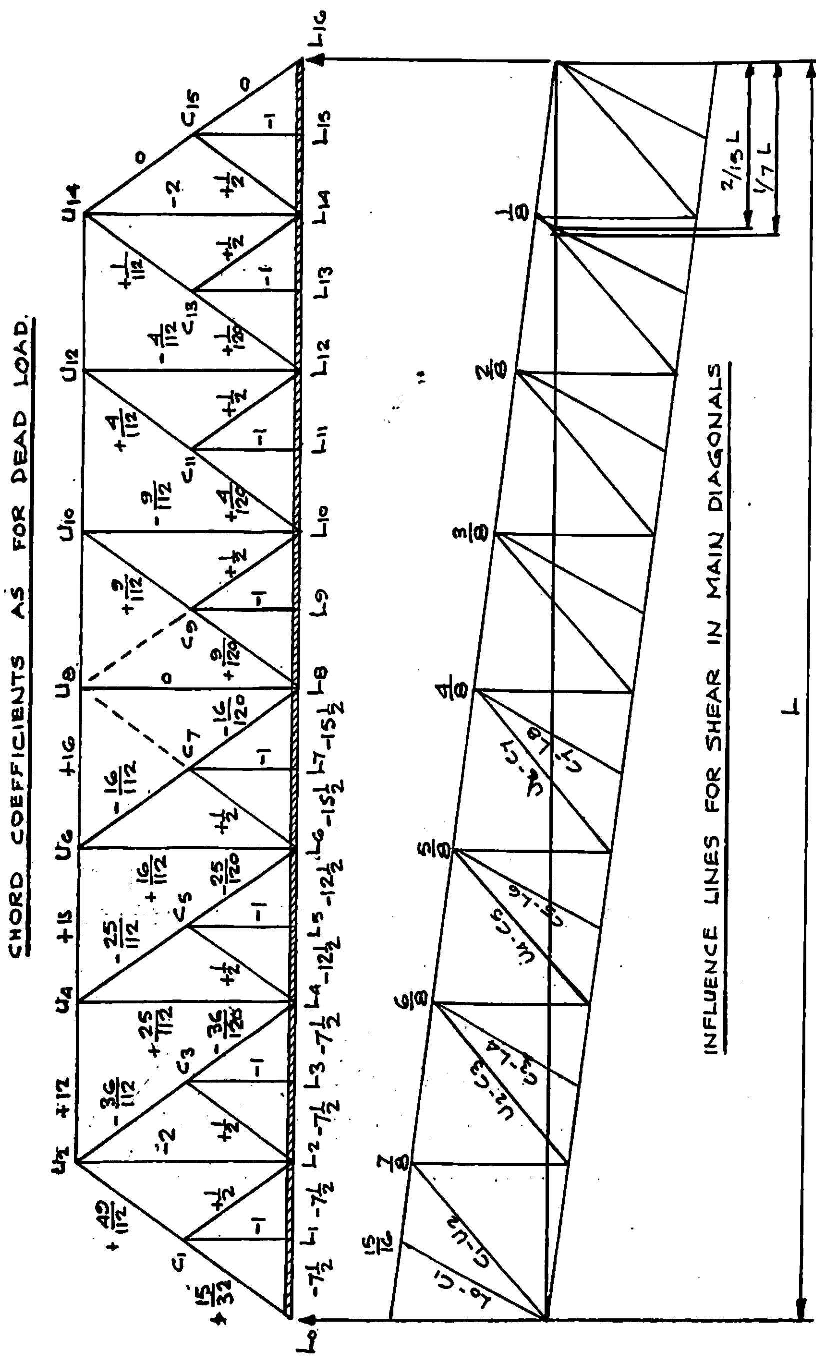


FIG. 27 - COEFFICIENTS FOR UNIFORMLY DISTRIBUTED LOAD FOR PRATT TRUSS WITH SUB-DIVIDED PANELS.

from the end would be $1 \times {}^{9}/_{10}$, $3 \times {}^{8}/_{10}$, $5 \times {}^{7}/_{10}$, $7 \times {}^{6}/_{10}$, and $9 \times {}^{5}/_{10}$, while the upper chord coefficients starting from the end would be $2 \times {}^{9}/_{10}$, $4 \times {}^{8}/_{10}$, $6 \times {}^{7}/_{10}$, $8 \times {}^{6}/_{10}$, and $10 \times {}^{5}/_{10}$.

Fig. 26 gives coefficients for a uniformly distributed load and for a single moving load for a Pratt truss, and after the detailed explanation given for Figs. 24 and 25 should be self-explanatory since the system of coefficient increments follows exactly the same rules.

Following upon the explanations given for Figs. 24 and 25, it is now possible to write down the coefficients for a uniformly distributed load and for a single moving load for some of the frames previously analysed for other loading conditions.

For the Warren girder deck span shown in Fig. 16 and taking first the case of a uniformly distributed load the coefficients for the chord members will be as shown in Fig. 16, since these values are for the span fully loaded. For the web members the denominator, with seven panels, will be $2 \times 7 \times 6 = 84$, and the numerators reading from right to left will be 0, 0, $+1$, -1, $+4$, -4, $+9$, -9, $+16$, -16, $+25$, -25, $+36$ and -36. Except for the end panel where the loads are based upon the span fully loaded these web coefficients give smaller loads than for the condition in Fig. 16 with live loads concentrated at the panel points.

For the single moving load the diagonal coefficients from right to left will be 0, 0, $+{}^{1}/_{7}$, $-{}^{1}/_{7}$, $+{}^{2}/_{7}$, $-{}^{2}/_{7}$, $+{}^{3}/_{7}$, $-{}^{3}/_{7}$, $+{}^{4}/_{7}$, $-{}^{4}/_{7}$, $+{}^{5}/_{7}$, $-{}^{5}/_{7}$, $+{}^{6}/_{7}$, $-{}^{6}/_{7}$. The top chords coefficients will be

U0-U1 $(1 \times {}^{6}/_{7}) = +{}^{6}/_{7}$
U1-U2 $(3 \times {}^{5}/_{7}) = +{}^{15}/_{7}$
U2-U3 $(5 \times {}^{4}/_{7}) = +{}^{20}/_{7}$
U3-U4 $(7 \times {}^{3}/_{7}) = +{}^{21}/_{7}$

The bottom chord coefficients will be

L1-L2 $-(2 \times {}^{6}/_{7}) = -{}^{12}/_{7}$
L2-L3 $-(4 \times {}^{5}/_{7}) = -{}^{20}/_{7}$
L3-L4 $-(6 \times {}^{4}/_{7}) = -{}^{24}/_{7}$

For the Warren girder with verticals shown in Fig. 17 and taking first a uniformly distributed load the chord coefficients will again be as in Fig. 17, and for the web members with 14 panels the denominator will be $2 \times 14 \times 13 = 364$. The numerators reading from right to left will be 0, $+1$, -4, $+9$, -16, $+25$, -36, $+49$, -64, $+81$, -100, $+121$, -144, and $+169$, and except for the end post these values will again give smaller loads than for the live load condition analysed in Fig. 17.

For the single moving load the diagonal coefficients from right to left will be 0, $+{}^{1}/_{14}$, $-{}^{2}/_{14}$, $+{}^{3}/_{14}$, $-{}^{4}/_{14}$, $+{}^{5}/_{14}$, $-{}^{6}/_{14}$, $+{}^{7}/_{14}$,

$-{}^{8}/_{14}$, $+{}^{9}/_{14}$, $-{}^{10}/_{14}$, $+{}^{11}/_{14}$, $-{}^{12}/_{14}$, and $+{}^{13}/_{14}$. The top chord coefficients will be

U1-U3 $(2 \times {}^{12}/_{14}) = +{}^{24}/_{14}$
U3-U5 $(4 \times {}^{10}/_{14}) = +{}^{40}/_{14}$
U5-U7 $(6 \times {}^{8}/_{14}) = +{}^{48}/_{14}$

The bottom chord coefficients will be

L0-L2 $-(1 \times {}^{13}/_{14}) = -{}^{13}/_{14}$
L2-L4 $-(3 \times {}^{11}/_{14}) = -{}^{33}/_{14}$
L4-L6 $= -(5 \times {}^{9}/_{14}) = -{}^{45}/_{14}$
L6-L8 $= -(7 \times {}^{7}/_{14}) = -{}^{49}/_{14}$

For the Warren girder deck span with verticals shown in Fig. 18, the coefficients for both a uniformly distributed load and a single moving load will be similar to those given above for Fig. 17, except that the signs for the web coefficients will be reversed, and the chord coefficients will also be reversed. That is to say, the coefficients for the top chord in Fig. 18 will have the same numerical value as these for the bottom chord in Fig. 17 with signs reversed, and the coefficients for the bottom chord in Fig. 18 will have the same numerical value as those for the top chord in Fig. 17 also with reversed signs.

For the Pratt truss with sub-divided panels, shown in Fig. 19, and for the Warren girder also with sub-divisions shewn in Fig. 20, the procedure for a uniformly distributed load and for a single moving load is general similar to previous examples varied only by the effect of the sub-verticals and sub-diagonals.

Taking first the Pratt truss with a uniform load this condition is set down in Fig. 27, which includes influence lines for shear in the web members in order to explain the effect of the sub-divisions. For the upper portion of the diagonals and for the main verticals the coefficients are the same as for a truss without sub-divisions (see Fig. 26) since the neutral points divide the span into seven equal parts. For the lower portions of the diagonals (excluding the end post) the geometry of the influence triangles fixes the position of the neutral points (for 16 panels) at intervals of two fifteenths of the span. Hence starting from the right the areas of the influence triangles are

$\frac{1}{2} \times {}^{2}/_{15} \times \frac{1}{8} \times L = {}^{1}/_{120}\,L$
$\frac{1}{2} \times {}^{4}/_{15} \times \frac{2}{8} \times L = {}^{4}/_{120}\,L$
$\frac{1}{2} \times {}^{6}/_{15} \times \frac{3}{8} \times L = {}^{9}/_{120}\,L$
$\frac{1}{2} \times {}^{8}/_{15} \times \frac{4}{8} \times L = {}^{16}/_{120}\,L$
$\frac{1}{2} \times {}^{10}/_{15} \times \frac{5}{8} \times L = {}^{25}/_{120}\,L$
$\frac{1}{2} \times {}^{12}/_{15} \times \frac{6}{8} \times L = {}^{36}/_{120}\,L$
$\frac{1}{2} \times {}^{14}/_{15} \times \frac{7}{8} \times L = {}^{49}/_{120}\,L$

In other words the numerators are the same as for the upper portions of the diagonals and the denominators for eight main panels and sixteen sub-panels are $8 \times 15 = 120$.

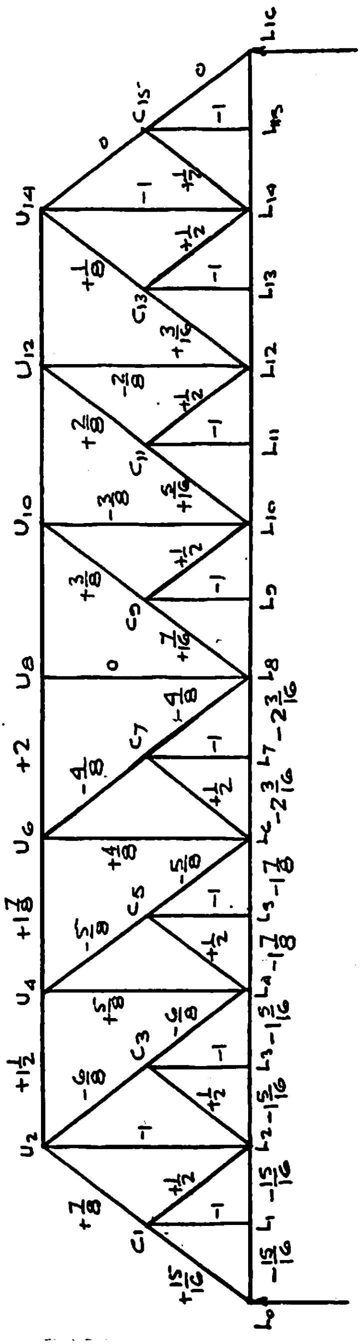


FIG. 28 - COEFFICIENTS FOR SINGLE MOVING LOAD FOR PRATT TRUSS WITH SUB-DIVIDED PANELS

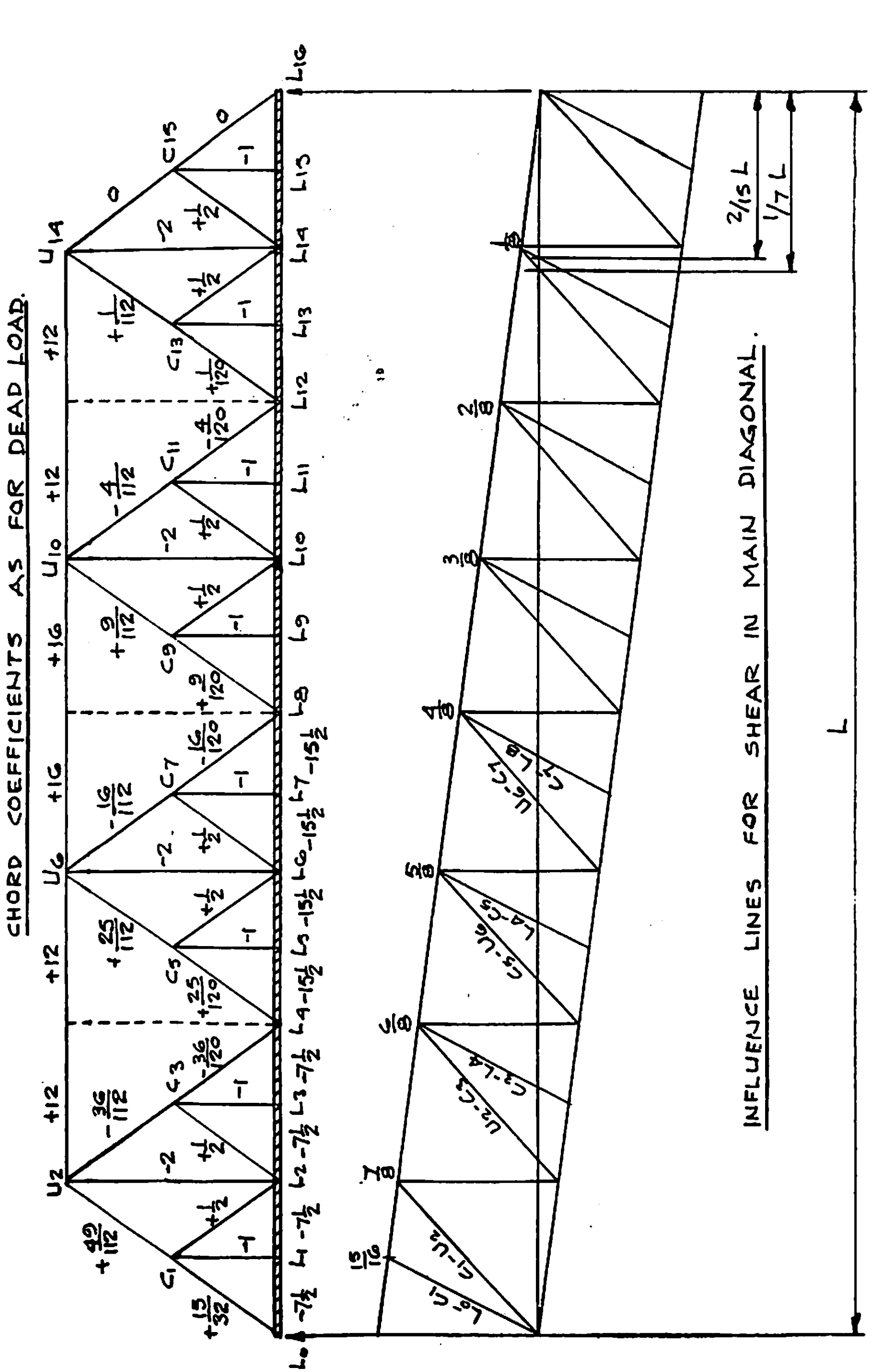


FIG. 29 - COEFFICIENTS FOR UNIFORMLY DISTRIBUTED LOAD FOR WARREN GIRDER.

For the lower portion of the end post it will be clear from the influence line that the area of the influence triangle will be

$$\tfrac{1}{2} \times {}^{15}/_{15} \times {}^{15}/_{16} \times L = {}^{15}/_{32}\, L$$

In other words the coefficient for this member will be half of the ordinate in the influence triangle.

From the above it will be clear that a general principle can again be established irrespective of the number of panels. For example, with ten main panels and twenty sub-panels the denominator for the upper portions of the main diagonals and for the main verticals would be $10 \times 9 \times 2 = 180$, and for the lower portions of the main diagonals $10 \times 19 = 190$, with numerators for all these members following the general sequence of 1^2, 2^2 - - - - - 9^2, while for the lower portion of the end post the coefficient would be $\tfrac{1}{2} \times {}^{19}/_{20} = {}^{19}/_{40}$.

The condition for a single moving load for this girder is given in Fig. 28 and after previous examples will be almost self-explanatory. Taking the diagonals first, maximum effect on L0-C1 occurs with the load at C1 and hence the coefficient for this member is $+{}^{15}/_{16}$, the shear in the end panel. Maximum effect on C1-U2 occurs with the load at L2, giving a coefficient of $+\tfrac{7}{8}$, the shear in the left-hand panel. This position of the load also gives a reversal of $+\tfrac{1}{8}$ in U2-C3, the shear in the right-hand panel and this value is placed on C13-U14. Maximum effect on U2-L4 occurs with the load at L4, giving a coefficient of $-\tfrac{6}{8}$, the shear in the left-hand panel. This position of the load gives a reversal of $-\tfrac{2}{8}$ in U4-L4 and of $+\tfrac{2}{8}$ in U4-C5, and these values are placed on U12-L12 and U12-C11 respectively. Similarly maximum effect on U4-L6 occurs with the load at L6, giving a coefficient of $-\tfrac{5}{8}$ with a reversal of $-\tfrac{3}{8}$ in U6-L6 and $+\tfrac{3}{8}$ in U6-C7, placed respectively on U10-L10 and U10-C9. In the same way the maximum coefficient for U6-L8 is $-\tfrac{4}{8}$ with the load at L8.

Loads at C3, C5 and C7 do not increase the main coefficients in the web members since part of the load is taken by the sub-diagonals and the effect upon the upper portions of the main diagonals is less than the effect with the load at L4, L6 and L8. The reversals for the lower portions of the main diagonals are, however, increased. With the load at C3 the shear to the right is $\tfrac{3}{16}$ and this gives a reversal of $+\tfrac{3}{16}$ for C3-L4, placed on C13-L12. With the load at C5 the shear to the right is $\tfrac{5}{16}$, giving a reversal of $+\tfrac{5}{16}$ in C5-L6, placed on C11-L10, and with the load at C7 the reversal in C7-L8 is $+\tfrac{7}{16}$, placed on C9-L8.

For the top chord maximum effect on U2-U4 occurs with the load at L4. This position gives a coefficient of $+\tfrac{6}{8}$ in L0-U2 and $-\tfrac{6}{8}$ in U2-L4, and the addition of these figures gives a coefficient of $+1\tfrac{1}{2}$ for U2-U4. Maximum effect on U4-U6 occurs with the load at L6, giving coefficients of $\tfrac{5}{8}$ for L0-U2, U2-L4, and U4-L6,

and the addition of these figures gives a coefficient of $+1\frac{7}{8}$ for U4-U6. Similarly maximum effect on U6-U8 occurs with the load at L8, giving a coefficient of $\frac{4}{8}$ for all diagonals to the left of L8, and the addition of these figures gives a coefficient of $+2$ for U6-U8.

For the lower chord maximum effect on L0-L2 occurs with the load at C1 which gives a coefficient of $+\frac{15}{16}$ for L0-C1 and hence $-\frac{15}{16}$ for L0-L2. Maximum effect on L2-L4 occurs with the load at C3 which gives coefficient of $+\frac{13}{16}$ in L0-U2 and $+\frac{1}{2}$ in C3-L2, and for cancellation across these two members the coefficient for L2-L4 must be $-(\frac{13}{16}+\frac{1}{2}) = -1\frac{5}{16}$. Maximum effect on L4-L6 occurs with the load at C5, which gives coefficients of $\frac{11}{16}$ in L0-U2 and U2-L4 (and hence a coefficient of $+1\frac{3}{8}$ in U2-U4) and a coefficient of $+\frac{1}{2}$ in C5-L4. Adding up across U2-U4 and C5-L4 gives a coefficient of $-1\frac{7}{8}$ for L4-L6. Maximum effect on L6-L8 occurs with the load at C7, which gives coefficients of $\frac{9}{16}$ for L0-U2, U2-L4, and U4-L6 (and hence a coefficient of $+{}^{27}/_{16}$ for U4-U6) and a coefficient of $+\frac{1}{2}$ for C7-L6. Adding up across U4-U6 and C7-L6 gives a coefficient of $-2\frac{3}{16}$ for L6-L8.

Taking next the case of a Warren girder with sub-divided panels Fig. 29 gives the coefficients for a uniformly distributed load and after the detailed explanation given for Fig. 27 this should be clear, the only variation being occasioned by the shape of the truss.

The case of a single moving load is given in Fig. 30, and this again follows generally the procedure given for Fig. 28, varied only by the shape of the truss. A load at C1 gives maximum coefficients for L0-C1 and for L0-L2. A load at L2 gives maximum coefficient for C1-U2, and a reversal in U2-C3 placed on U14-C13. A load at C3 gives maximum coefficient for L2-L4 and a reversal in C3-L4 placed on C13-L12. A load at L4 gives maximum coefficients for U2-L4 and for U2-U4 and a reversal in L4-U6 placed on L12-U10. A load at C5 gives maximum coefficients for C5-L4 and for L4-L6, but no reversal to the right owing to the effect of sub-diagonal C5-L6. A load at L6 gives maximum coefficients for U6-C5 and a reversal in U6-C7 placed on U10-C9. A load at C7 gives maximum coefficient in L6-L8 and a reversal in C7-L8 placed on C9-L8. A load at L8 gives maximum coefficients for U6-L8 and for U6-U10.

For the K-truss shown in Fig. 21, the coefficients for a uniformly distributed load and for a single moving load are given in Figs. 31 and 32 respectively. Taking first the case of the uniform load given in Fig. 31, the influence lines for shear indicate the method of calculation. With a double bracing system the diagonals in each panel from line 2 to line 10 share the shear, and for these members the denominator will therefore be $12 \times 11 \times 2 \times 2 = 528$, and the numerators will be 2^2, 3^2, 4^2 - - - - - 9^2. The second panel from the right, being part of the end post system, has no

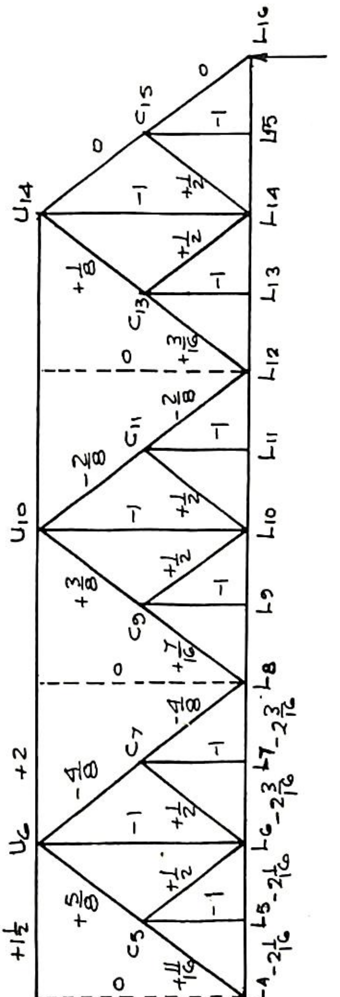


FIG. 30- COEFFICIENTS FOR SINGLE MOVING LOAD FOR WARREN GIRDER WITH SUB-DIVIDED PANELS.

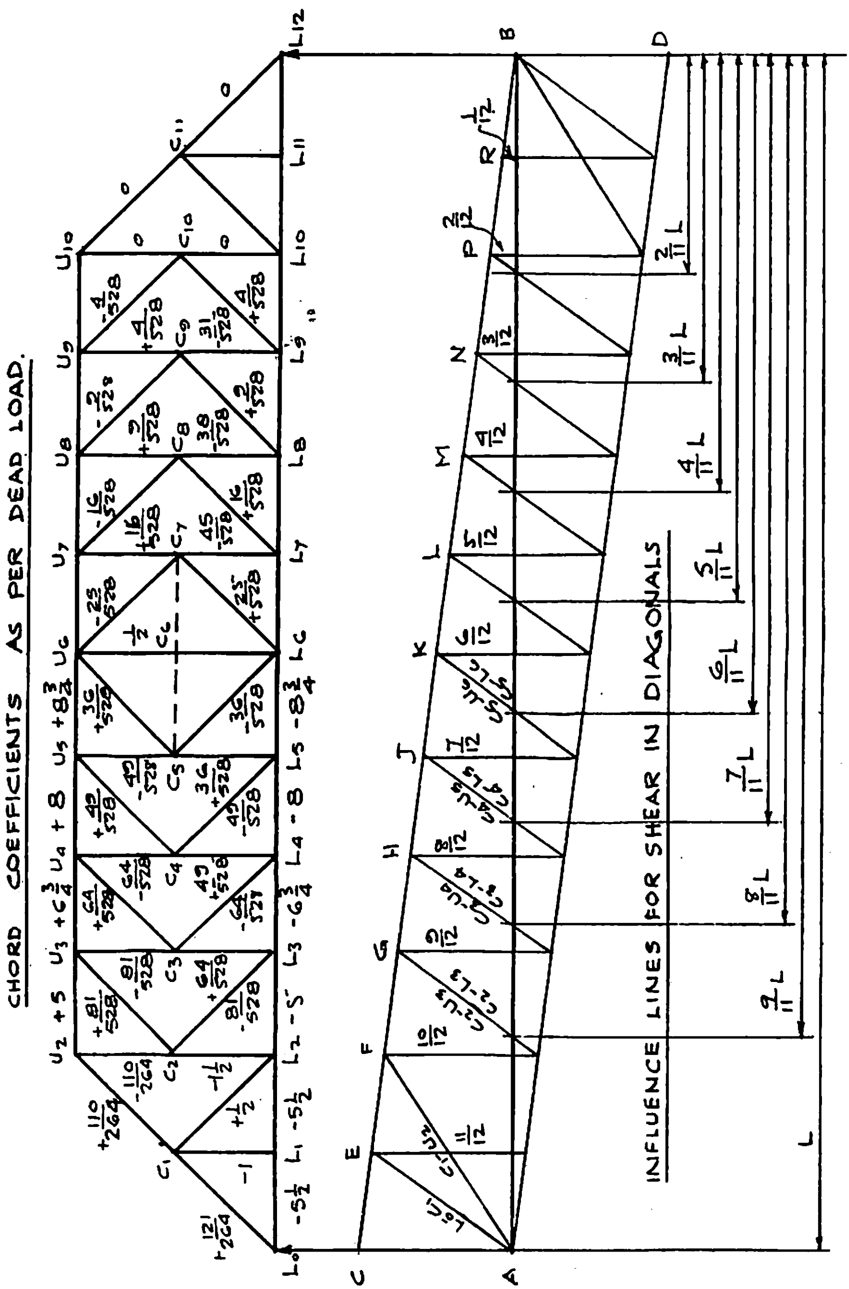


FIG. 31 - COEFFICIENTS FOR UNIFORMLY DISTRIBUTED LOAD FOR K-TRUSS.

reversal. The upper verticals on both halves of the girder will equilibrate the vertical components of the diagonals, meeting them at the upper chord panel points, and the lower verticals on the left-hand half of the girder will equilibrate the diagonals meeting them at the centre intersection points as in the case of Fig. 21. The coefficients for the lower verticals on the right hand half of the girder (giving the reversals for corresponding members on the left) will be the difference between the maximum coefficients and the dead load coefficients given in Fig. 21. The coefficients for the chord members will be the same as for dead load, that is, assuming the girder to be fully loaded with loads concentrated at the panel points.

For the single moving load, shown in Fig. 32, a load placed at C1 gives maximum coefficients for L0-C1 and for L0-L2 equal to the shear in the end panel, and also a coefficient of $+\frac{1}{2}$ for C1-L2. A load at L2 gives a coefficient of -1 for L2-C2, which is merely a hanger for this condition. At C2 this load divides according to the shear, $^{10}/_{12}$ passing through C2-U2 into U2-C1, and $^{2}/_{12}$ being shared equally between C2-U3 and C2-L3, giving the reversals in these members which are placed on C10-U9 and C10-L9. This position of the load also gives a maximum coefficient of $^{10}/_{12}$ for U2-U3 and for L2-L3. With the load at L3 the shear of $^{9}/_{12}$ to the left is shared by C2-U3 and C2-L3, and the shear of $^{3}/_{12}$ to the right is shared by C3-U4 and C3-L4, giving the reversals for these members which are placed on C9-U8 and C9-L8. Lower vertical C3-L3 must equilibrate C2-L3 and the load at L3, and this fixes its coefficient of $-^{15}/_{24}$. This position of the load gives maximum coefficients for U3-U4 and L3-L4 which must be $^{9}/_{12}+^{9}/_{24}$ since the coefficient for L0-U2 is $^{9}/_{12}$ for this position of the load.

With the load at L4, shear to the left is $^{8}/_{12}$ and to the right $^{4}/_{12}$, giving coefficients of $^{8}/_{24}$ for C3-U4 and C3-L4 and reversals of $^{4}/_{24}$ for C4-U5 and C4-L5 which are placed on C8-U7 and C8-L7. Vertical C4-L4 must equilibrate C3-L4 and the load at L4 and this fixes its coefficient of $-^{16}/_{24}$. For this position of the load the coefficients for chord members U4-U5 and L4-L5 will be $^{8}/_{12}+^{8}/_{24}+^{8}/_{24} = ^{32}/_{24}$ on the same principle as for U3-U4 and L3-L4 above. This position of the load will give the reversal for vertical C3-L3 which will be $+^{8}/_{24}$ to equilibrate C3-L4, and this reversal is placed on corresponding member C9-L9.

With the load at L5, giving a left-hand shear of $^{7}/_{12}$ and a right-hand shear of $^{5}/_{12}$, the coefficients for C4-U5 and C4-L5 are $^{7}/_{24}$ and reversals for C5-U6 and C5-L6 are $^{5}/_{24}$ placed on members C7-U6 and C7-L6. Vertical C5-L5 has a coefficient of $-^{17}/_{24}$ to equilibrate C4-L5 and the load at L5, and in the same way as before chord members U5-U6 and L5-L6 have coefficients of $^{7}/_{12}+^{7}/_{24}+^{7}/_{24}+^{7}/_{24} = ^{35}/_{24}$. This position of the load gives the reversal

for vertical C4-L4, which will be $+{}^{7}/_{24}$ to equilibrate C4-L5 and this reversal is placed on corresponding member C8-L8.

With the load at L6, half will go each way giving coefficients of $-{}^{6}/_{24}$ in the lower diagonals and $+{}^{6}/_{24}$ in the upper diagonals with a coefficient of $-{}^{6}/_{12}$ in the centre vertical for equilibrium. This position of the load gives the reversal for vertical C5-L5 which will be $+{}^{6}/_{24}$ to equilibrate C5-L6, and this reversal is placed on member C7-L6.

The upper verticals in all cases will equilibrate the diagonals meeting them at the upper chord panel points, the values on the right-hand half being the reversals for the corresponding members on the left.

Fig. 33 shows a type of girder often used on the Continent but rarely employed in this country, in which the loads are suspended from the centre intersection points instead of from the lower chord. Dead load coefficients follow standard practice, the load at C4 being shared by C4-U3 and C4-L3, and thence passing through U3-C3 and L3-C3 into the next panel, and so on to the reaction, being joined by the loads at C3 and C2 en route. The chord coefficients again add up across the diagonals. The live loads are assumed to act at the panel points, and with seven panels of loading the web coefficients starting from the right are 0, ${}^{1}/_{7}$, ${}^{3}/_{7}$, ${}^{6}/_{7}$, ${}^{10}/_{7}$, ${}^{15}/_{7}$ and ${}^{21}/_{7}$, the coefficients on the right-hand half being the reversals for the corresponding members on the left-hand half. As in the case of the K-truss the upper and lower diagonals in each panel will share these coefficients equally. The chord coefficients will be the same as for dead load with the girder fully loaded.

In order to illustrate the method of analysis and the tabulation of loads a typical example is given in Fig. 34 for a Warren girder supporting concentrated live loads and a knife edge load, generally along the lines required for the British Standard and Ministry of Transport Standard loading for highway bridges, and in order to show how small is the difference between the effect of concentrated live loads and uniformly distributed loads an alternative analysis is included for the latter condition.

No explanation should be necessary regarding the shear coefficients, and the length coefficients will be functions of the angle of slope of the diagonals.

Length coefficient K_L for diagonals = co-secant 60° = 1·155.

" " " chords = co-tangent 60° = 0·5775.

In order to simplify calculation the length coefficient will be combined with the panel point load P.

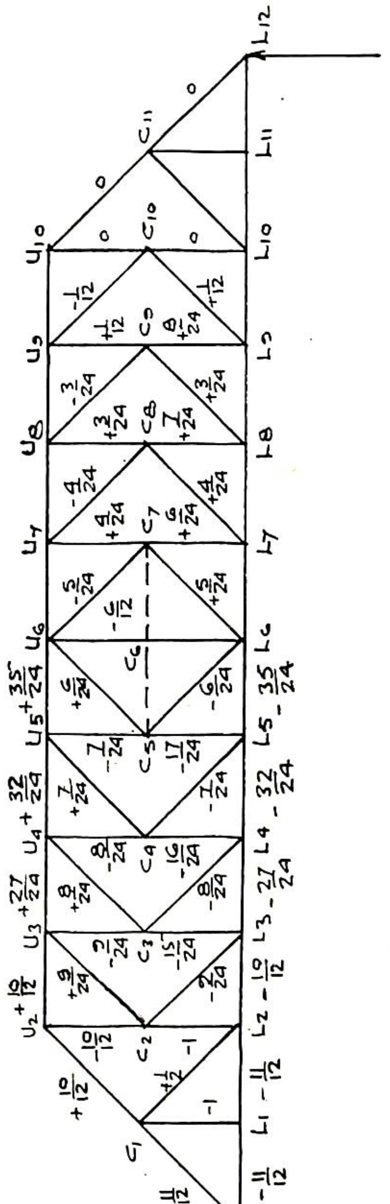


FIG. 32 - COEFFICIENTS FOR SINGLE MOVING LOAD FOR K- TRUSS.

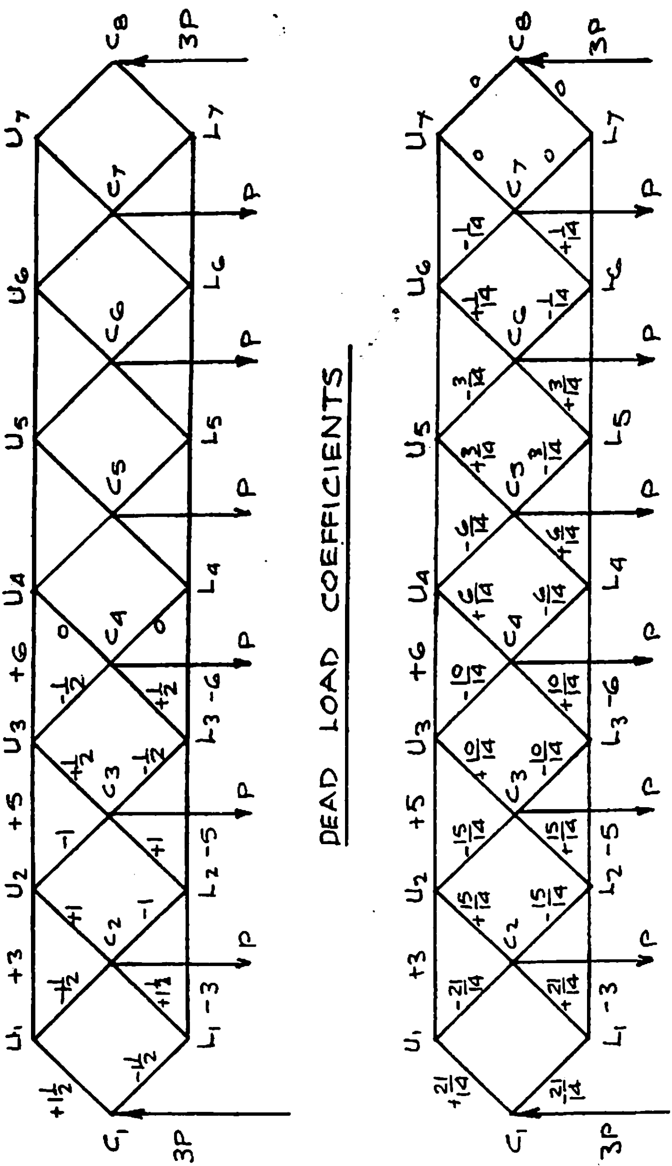


FIG. 33 - DEAD LOAD & LIVE LOAD COEFFICIENTS FOR DOUBLE BRACED GIRDER WITHOUT VERTICALS

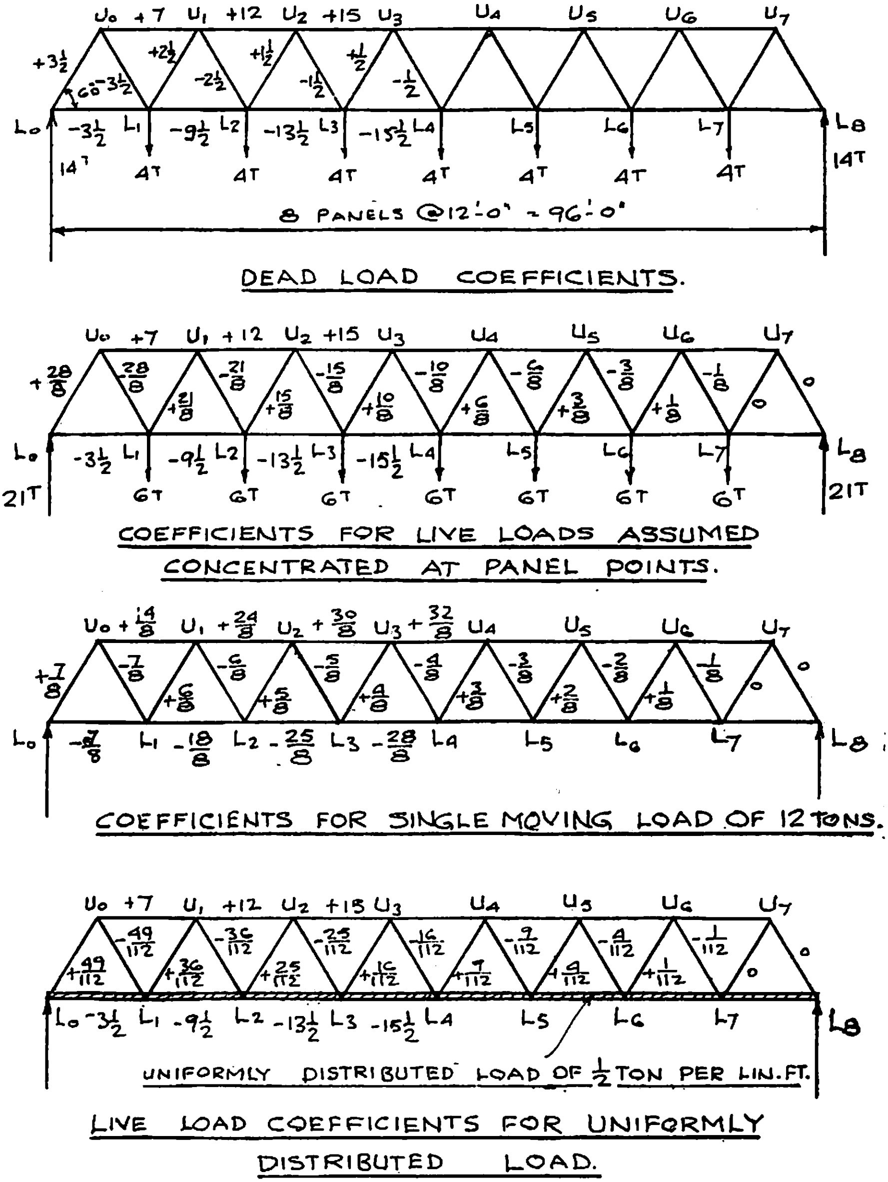


FIG. 34 - TYPICAL EXAMPLE SHOWING METHOD OF PROCEDURE.

Table of Loads for Warren Girder in Fig. 34 (based on Concentrated Live Loads) with alternative Loads for U.D.L.L. ALL LOADS IN TONS

Member	Dead Load	Concentrated Live Load	Single Moving Load	Design Load	Alternative Live Load for U.D. Condition
L_0-L_1	$-(3\frac{1}{2}\times 2{\cdot}31)=-8{\cdot}1$	$-(3\frac{1}{2}\times 3{\cdot}47)=-12{\cdot}1$	$-(\frac{7}{8}\times 6{\cdot}93)=-6{\cdot}1$	$-26{\cdot}3$	as for Concentrated Live Load
L_1-L_2	$-(9\frac{1}{2}\times 2{\cdot}31)=-21{\cdot}9$	$-(9\frac{1}{2}\times 3{\cdot}47)=-32{\cdot}9$	$-(\frac{18}{8}\times 6{\cdot}93)=-15{\cdot}6$	$-70{\cdot}4$	
L_2-L_3	$-(13\frac{1}{2}\times 2{\cdot}31)=-31{\cdot}2$	$-(13\frac{1}{2}\times 3{\cdot}47)=-46{\cdot}8$	$-(\frac{25}{8}\times 6{\cdot}93)=-21{\cdot}7$	$-99{\cdot}7$	
L_3-L_4	$-(15\frac{1}{2}\times 2{\cdot}31)=-35{\cdot}8$	$-(15\frac{1}{2}\times 3{\cdot}47)=-53{\cdot}7$	$-(\frac{28}{8}\times 6{\cdot}93)=-24{\cdot}3$	$-113{\cdot}8$	
U_0-U_1	$+(7\times 2{\cdot}31)=+16{\cdot}2$	$+(7\times 3{\cdot}47)=+24{\cdot}3$	$+(\frac{14}{8}\times 6{\cdot}93)=+12{\cdot}1$	$+52{\cdot}6$	
U_1-U_2	$+(12\times 2{\cdot}31)=+27{\cdot}7$	$+(12\times 3{\cdot}47)=+41{\cdot}6$	$+(\frac{24}{8}\times 6{\cdot}93)=+20{\cdot}8$	$+90{\cdot}1$	
U_2-U_3	$+(15\times 2{\cdot}31)=+34{\cdot}7$	$+(15\times 3{\cdot}47)=+52{\cdot}0$	$+(\frac{30}{8}\times 6{\cdot}93)=+26{\cdot}0$	$+112{\cdot}7$	
U_3-U_4	$+(16\times 2{\cdot}31)=+37{\cdot}0$	$+(16\times 3{\cdot}47)=+55{\cdot}5$	$+(\frac{32}{8}\times 6{\cdot}93)=+27{\cdot}7$	$+120{\cdot}2$	
L_0-U_0	$+(3\frac{1}{2}\times 4{\cdot}62)=+16{\cdot}2$	$+(\frac{28}{8}\times 6{\cdot}93)=+24{\cdot}3$	$+(\frac{7}{8}\times 13{\cdot}86)=+12{\cdot}2$	$+52{\cdot}7$	$+(\frac{49}{112}\times 55{\cdot}44)=+24{\cdot}3$
U_0-L_1	$-(3\frac{1}{2}\times 4{\cdot}62)=-16{\cdot}2$	$-(\frac{28}{8}\times 6{\cdot}93)=-24{\cdot}3$	$-(\frac{7}{8}\times 13{\cdot}86)=-12{\cdot}2$	$-52{\cdot}7$	$-(\frac{49}{112}\times 55{\cdot}44)=-24{\cdot}3$
L_1-U_1	$+(2\frac{1}{2}\times 4{\cdot}62)=+11{\cdot}6$	$+(\frac{21}{8}\times 6{\cdot}93)=+18{\cdot}2$	$+(\frac{6}{8}\times 13{\cdot}86)=+10{\cdot}4$	$+40{\cdot}2$	$+(\frac{36}{112}\times 55{\cdot}44)=+17{\cdot}8$
L_1-U_1 (reversal)	$+(2\frac{1}{2}\times 4{\cdot}62)=+11{\cdot}6$	$-(\frac{1}{8}\times 6{\cdot}93)=-0{\cdot}9$	$-(\frac{1}{8}\times 13{\cdot}86)=-1{\cdot}7$	NIL	$-(\frac{1}{112}\times 55{\cdot}44)=-0{\cdot}5$
U_1-L_2	$-(2\frac{1}{2}\times 4{\cdot}62)=-11{\cdot}6$	$-(\frac{21}{8}\times 6{\cdot}93)=-18{\cdot}2$	$-(\frac{6}{8}\times 13{\cdot}86)=-10{\cdot}4$	$-40{\cdot}2$	$-(\frac{36}{112}\times 55{\cdot}44)=-17{\cdot}8$
U_1-L_2 (reversal)	$-(2\frac{1}{2}\times 4{\cdot}62)=-11{\cdot}6$	$+(\frac{1}{8}\times 6{\cdot}93)=+0{\cdot}9$	$+(\frac{1}{8}\times 13{\cdot}86)=+1{\cdot}7$	NIL	$+(\frac{1}{112}\times 55{\cdot}44)=+0{\cdot}5$
L_2-U_2	$+(1\frac{1}{2}\times 4{\cdot}62)=+6{\cdot}9$	$+(\frac{15}{8}\times 6{\cdot}93)=+13{\cdot}0$	$+(\frac{5}{8}\times 13{\cdot}86)=+8{\cdot}7$	$+28{\cdot}6$	$+(\frac{25}{112}\times 55{\cdot}44)=+12{\cdot}4$
L_2-U_2 (reversal)	$+(1\frac{1}{2}\times 4{\cdot}62)=+6{\cdot}9$	$-(\frac{3}{8}\times 6{\cdot}93)=-2{\cdot}6$	$-(\frac{3}{8}\times 13{\cdot}86)=-3{\cdot}5$	NIL	$-(\frac{4}{112}\times 55{\cdot}44)=-2{\cdot}0$
U_2-L_3	$-(1\frac{1}{2}\times 4{\cdot}62)=-6{\cdot}9$	$-(\frac{15}{8}\times 6{\cdot}93)=-13{\cdot}0$	$-(\frac{5}{8}\times 13{\cdot}86)=-8{\cdot}7$	$-28{\cdot}6$	$-(\frac{25}{112}\times 55{\cdot}44)=-12{\cdot}4$
U_2-L_3 (reversal)	$-(1\frac{1}{2}\times 4{\cdot}62)=-6{\cdot}9$	$+(\frac{3}{8}\times 6{\cdot}93)=+2{\cdot}6$	$+(\frac{3}{8}\times 13{\cdot}86)=+3{\cdot}5$	NIL	$+(\frac{4}{112}\times 55{\cdot}44)=+2{\cdot}0$
L_3-U_3	$+(\frac{1}{2}\times 4{\cdot}62)=+2{\cdot}3$	$+(\frac{10}{8}\times 6{\cdot}93)=+8{\cdot}7$	$+(\frac{4}{8}\times 13{\cdot}86)=+6{\cdot}9$	$+17{\cdot}9$	$+(\frac{16}{112}\times 55{\cdot}44)=+7{\cdot}9$
L_3-U_3 (reversal)	$+(\frac{1}{2}\times 4{\cdot}62)=+2{\cdot}3$	$-(\frac{6}{8}\times 6{\cdot}93)=-5{\cdot}2$	$-(\frac{3}{8}\times 13{\cdot}86)=-5{\cdot}2$	$-8{\cdot}1$	$-(\frac{9}{112}\times 55{\cdot}44)=-4{\cdot}5$
U_3-L_4	$-(\frac{1}{2}\times 4{\cdot}62)=-2{\cdot}3$	$-(\frac{10}{8}\times 6{\cdot}93)=-8{\cdot}7$	$-(\frac{4}{8}\times 13{\cdot}86)=-6{\cdot}9$	$-17{\cdot}9$	$-(\frac{16}{112}\times 55{\cdot}44)=-7{\cdot}9$
U_3-L_4 (reversal)	$-(\frac{1}{2}\times 4{\cdot}62)=-2{\cdot}3$	$+(\frac{6}{8}\times 6{\cdot}93)=+5{\cdot}2$	$+(\frac{3}{8}\times 13{\cdot}86)=+5{\cdot}2$	$+8{\cdot}1$	$+(\frac{9}{112}\times 55{\cdot}44)=+4{\cdot}5$

For dead load	$K_L \times P$ for diagonals	$= 1{\cdot}155 \times 4$	=	$4{\cdot}62$
	$K_L \times P$ for chords	$= 0{\cdot}5775 \times 4$	=	$2{\cdot}31$
For live load	$K_L \times P$ for diagonals	$= 1{\cdot}155 \times 6$	=	$6{\cdot}93$
	$K_L \times P$ for chords	$= 0{\cdot}5775 \times 6$	=	$3{\cdot}47$
For single load	$K_L \times P$ for diagonals	$= 1{\cdot}155 \times 12$	=	$13{\cdot}86$
	$K_L \times P$ for chords	$= 0{\cdot}5775 \times 12$	=	$6{\cdot}93$

The loads in the members are set out in the accompanying table. Slide rule accuracy is invariably sufficient for calculation, since in most cases it is sufficiently accurate to obtain the design load to the nearest whole number. By setting the slide rule at each of the constants in the table in turn the labour of calculation is very small especially as many of the loads in each column are related to each other.

In the case of the diagonals where the reversals are stated to be nil, this is because the effect of the dead load is greater than the combined effect of the live load and single load reversal, and in this girder only the centre diagonals have a design load reversal. This is fairly general in short span girders unless the live load is exceptionally high, but in longer spans several of the diagonals will have design reversals.

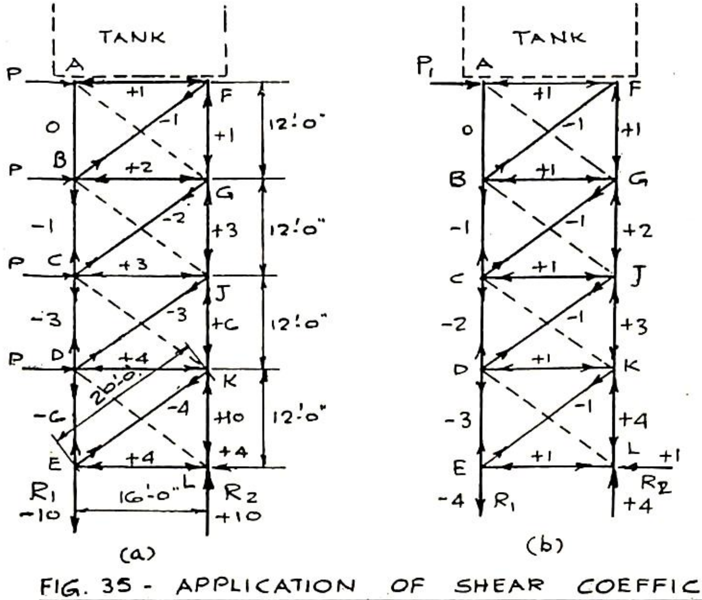


FIG. 35 - APPLICATION OF SHEAR COEFFICIENTS TO BRACED TOWER

For the alternative condition of a uniformly distributed live load the total load on the girder is $\frac{1}{2} \times 96 = 48$ tons, and the combined length and load coefficient for the diagonals will therefore be $1{\cdot}155 \times 48 = 55{\cdot}44$.

Shear coefficients can be employed for the analysis of any type of braced frame which lends itself to the procedure. Fig. 35 shows the method of finding the forces in the members of a braced tower due to the effect of wind pressure. The wind load on the tank is transferred to the top panel point for the purpose of analysis, the excess wind load at this point being dealt with separately. Diagram (*a*) gives the effect of equal panel point loads and diagram (*b*) the effect of the excess load. The procedure should be self-explanatory, the tower legs corresponding to the chords in previous examples. Reaction R_2 equals the load in member K-L, and reaction R_1 equilibrates members D-E and E-K.

For any one portion of the tower legs (say G-J) the load due to wind will be

$$+ (3 \times P \times {}^{12}/_{16}) + (2 \times P_1 \times {}^{12}/_{16}.$$

For any diagonal (say G-C) the load will be

$$- \{ (2 \times P \times {}^{20}/_{16}) + (1 \times P_1 \times {}^{20}/_{16}) \}.$$

This is a typical example and it will be clear that any braced frame with parallel chords and loads acting at right angles to the chords can be analysed in the same manner.

ACKNOWLEDGMENT.

The author wishes to acknowledge with thanks assistance received from Mr. P. T. Patel, B.E., of Braithwaite & Co. Structural Ltd, in checking the manuscript and preparation of diagrams.

BIBLIOGRAPHY.

MILO S. KETCHUM. *The Design of Highway Bridges.*

A.E.S.D. Printed Pamphlets and Other Publications in Stock.

An up-to-date list of A.E.S.D. pamphlets in stock is obtainable on application to the Editor, *The Draughtsman*, Onslow Hall, Little Green, Richmond, Surrey.

A similar list is also published in *The Draughtsman* twice a year.

Readers are asked to consult this list before ordering pamphlets published in previous sessions.

LIST OF A.E.S.D. DATA SHEETS.

1.	Safe Load on Machine-Cut Spur Gears.	
2.	Deflection of Shafts and Beams.	Connected.
3.	Deflection of Shafts and Beams (Instruction Sheet).	
4.	Steam Radiation Heating Chart.	
5.	Horse-Power of Leather Belts, etc.	
6.	Automobile Brakes (Axle Brakes).	Connected.
7.	Automobile Brakes (Transmission Brakes).	
8.	Capacities of Bucket Elevators.	
9.	Valley Angle Chart for Hoppers and Chutes.	
10.	Shafts up to 5½ inch diameter, subjected to Twisting and Combined Bending and Twisting.	
11.	Shafts, 5¾ to 26 inch diameter, subjected to Twisting and Combined Bending and Twisting.	
12.	Ship Derrick Booms.	
13.	Spiral Springs (Diameter of Round or Square Wire).	
14.	Spiral Springs (Compression).	
15.	Automobile Clutches (Cone Clutches).	
16.	,, ,, (Plate Clutches).	
17.	Coil Friction for Belts, etc.	
18.	Internal Expanding Brakes. Self-Balancing Brake Shoes (Force Diagram).	Connected
19.	Internal Expanding Brakes. Angular Proportions for Self-Balancing.	
20.	Referred Mean Pressure Cut-Off, etc.	
21.	Particulars for Balata Belt Drives.	
22.	⅞″ Square Duralumin Tubes as Struts.	
23.	1″ ,, ,, ,, ,,	
24.	¾″ Square Steel Tubes as Struts (30 ton yield).	
25.	⅞″ ,, ,, ,, (30 ton yield).	
26.	1″ ,, ,, ,, (30 ton yield).	
27.	¾″ ,, ,, ,, (40 ton yield).	
28.	⅞″ ,, ,, ,, (40 ton yield).	
29.	1″ ,, ,, ,, (40 ton yield).	
30.	Moments of Inertia of Built-up Sections (Tables).	Connected.
31.	Moments of Inertia of Built-up Sections (Instructions and Examples).	
34.	Capacity and Speed Chart for Troughed Band Conveyors.	
35.	Screw Propeller Design (Sheet 1, Diameter Chart).	Connected.
36.	,, ,, ,, (Sheet 2, Pitch Chart).	
37.	,, ,, ,, (Sheet 3, Notes & Examples).	
38.	Open Coil Conical Springs.	
39.	Close Coil Conical Springs.	
40.	Trajectory Described by Belt Conveyors (Revised, 1949).	
41.	Metric Equivalents.	
42.	Useful Conversion Factors.	
43.	Torsion of Non-Circular Shafts.	
44.	Railway Vehicles on Curves.	
46.	Coned Plate Development.	
47.	Solution of Triangles (Sheet 1, Right Angles).	
48.	Solution of Triangles (Sheet 2, Oblique Angles).	
49.	Relation between Length, Linear Movement and Angular Movement of Lever (Diagram and Notes).	
50.	,, ,, ,, ,, ,, ,, (Chart).	
51.	Helix Angle and Efficiency of Screws and Worms.	
52.	Approximate Radius of Gyration of Various Sections.	
53.	Helical Spring Graphs (Round Wire.	Connected.
54.	,, ,, ,, (Round Wire).	
55.	,, ,, ,, (Square Wire).	

56. Relative Value of Welds to Rivets.
58. Graphs for Strength of Rectangular Flat Plates of Uniform Thickness.
59. ,, Deflection ,, ,, ,, ,, ,,
61. Deflection of Leaf Spring.
62. Strength of Leaf Spring.
63. Chart Showing Relationship of Various Hardness Tests.
64. Shaft Horse-Power and Proportions of Worm Gear.
65. Ring with Uniform Internal Load (Tangential Strain). Connected.
66. ,, ,, ,, ,, ,, (Tangential Stress).
67. Hub Pressed on to Steel Shaft. (Maximum Tangential Stress at Bore of Hub).
68. Hub Pressed on to Steel Shaft. (Radial Gripping Pressure between Hub and Shaft).
69. Rotating Disc (Steel) Tangential Strain. Connected.
70. ,, ,, ,, Stress.
71. Ring with Uniform External Load, Tangential Strain. Connected.
72. ,, ,, ,, ,, ,, Stress.
73. Viscosity Temperature Chart for Converting Commercial to Absolute Viscosities. Connected.
74. Journal Friction on Bearings.
75. Ring Oil Bearings.
76. Shearing and Bearing Values for High Tensile Structural Steel Shop Rivets in accordance with B.S.S. No. 548/1934.
78. Velocity of Flow in Pipes for a Given Delivery. Connected.
79. Delivery of Water in Pipes for a Given Head.
80. (See No. 105).
81. Involute Toothed Gearing Chart.
83. Variation of Suction Lift and Temperature for Centrifugal Pumps.
89. Curve Relating Natural Frequency and Deflection. Connected.
90. Vibration Transmissibility Curved or Elastic Suspension.
91. Instructions and Examples in the Use of Data Sheets, Nos. 89 and 90.
92. Pressure on Sides of Bunker.
93-4-5-6-7. Rolled Steel Sections.
98-9-100. Boiler Safety Valves.
102. Pressure Required for Blanking and Piercing.
103. Punch and Die Clearances for Blanking and Piercing.
104. Nomograph for Valley Angles of Hoppers and Chutes.
105. Permissible Working Stresses in Mild Steel Struts with B.S. 449, 1948.
106. Compound Cylinder (Similar Material) Radial Pressure of Common Diameter (D1).
107. True Angles for Pipe Bends.
108. Development of Spiral Chutes.
109. Vertical Pitch of Panels in a Frame with Bracing at a Constant Slope.
110. Permissible Stresses in Cranes—B.S. 2573. Table 1—Values of A & B Tons/Sq. In.
111. Permissible Stresses in Cranes—B.S. 2573. Table 2—Values of A & B Tons/Sq. In.
112. Permissible Stresses in Cranes—B.S. 2573. Table 3—Compressive Stresses in Axially Loaded Struts.
113. Permissible Stresses in Cranes—B.S. 2573. Table 4—Power Driven Rivets and Fit Bolts.
114. Permissible Stresses in Cranes—B.S. 2573. Table 5—Power Driven Rivets and Fit Bolts.
115. Rectangular Weld Groups with Eccentric Load in Plane of Weld.

(Data Sheets are 3d to Members, 6d to others, post free).

Orders for Pamphlets and Data Sheets to be sent to the Editor, *The Draughtsman*, cheques and orders being crossed "A.E.S.D."

ADVICE TO INTENDING AUTHORS OF A.E.S.D. PRINTED PAMPHLETS.

Pamphlets submitted to the National Technical Sub-Committee for consideration with a view to publication in this series should not exceed 10,000 to 15,000 words and about 20 illustrations. The aim should be the presentation of the subject clearly and concisely, avoiding digressions and redundance. Manuscripts are to be written in the third person. Copies of an article entitled "Hints on the Writing of Technical Articles" can be obtained from the Editor of *The Draughtsman.*

Drawings for illustrations should be done either on a good plain white paper or tracing cloth, deep black Indian ink being used. For ordinary purposes they should be made about one-and-a-half times the intended finished size, and it should be arranged that wherever possible these shall not be greater than a single full page of the pamphlet, as folded pages are objectionable, although, upon occasion, unavoidable. Where drawings are made larger, involving a greater reduction, the lines should be made slightly heavier and the printing rather larger than normal, as the greater reduction tends to make the lines appear faint and the printing excessively small in the reproduction. In the case of charts or curves set out on squared paper, either all the squares should be inked in, or the chart or curve should be retraced and the requisite squares inked in. Figures should be as self-evident as possible. Data should be presented in graphical form. Extensive tabular matter, if unavoidable, should be made into appendices.

Authors of pamphlets are requested to adhere to the standard symbols of the British Standards Institution, where lists of such standard symbols have been issued, as in the case of the electrical and other industries, and also to the *British Standard Engineering Symbols and Abbreviations*, No. 1991, published by the B.S.I. at 6/-. Attention might also be given to mathematical notation, where alternative methods exist, to ensure the minimum trouble in setting up by the printer.

The value of the pamphlet will be enhanced by stating where further information on the subject can be obtained. This should be given in the form of footnotes or a bibliography, including the name and initials of the author, title, publisher, and year of publication. When periodicals are referred to, volume and page also should be given. References should be checked carefully.

Manuscripts, in the first instance should be submitted to the Editor, *The Draughtsman*, Onslow Hall, Little Green, Richmond, Surrey.

For pamphlets, a grant of £30 will be made to the author, but special consideration will be given in the case of much larger pamphlets which may involve more than the usual amount of preparation.

Milne, Tannahill & Methven
Printers Ltd
12-14 Mill Street Perth